Period : 1

Eigen values problem arising from Population models :- (leslie models)

1, It is the method for representing dynamics of age (or) size structure combination.

2, It combines the population process is (birth and death). into a single model.

3, By convergent, the use only female part of population.

Problems:-

Q. The leslie model describes age specified Population growth as follows. Let the oldest age attained by the females in some animal Population be six years. Divide the population into 3 age classes of 2 years each. Let the leslie matrix be $L = [l_{jk}]$ which is equal to

$$\begin{bmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}$$

A) what is the number of the females in each class. After 2, 4, 6 years - If each class initially consist of 500 females?

B) for what initial distribution will number of females in each class change by same proportion. what is the rate of change.

Soln:- Initially $X_0 = \begin{pmatrix} 500 \\ 500 \\ 500 \end{pmatrix}$

After 2 years, the no. of females in each class

$$X_2 = LX_0 = \begin{pmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{pmatrix}_{3\times3} \begin{pmatrix} 500 \\ 500 \\ 500 \end{pmatrix}_{3\times1}$$

$$= \begin{pmatrix} 1350 \\ 300 \\ 150 \end{pmatrix}$$

$$X_4 = LX_2 = \begin{bmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix} \begin{pmatrix} 1350 \\ 300 \\ 150 \end{pmatrix} = \begin{pmatrix} 750 \\ 810 \\ 90 \end{pmatrix}$$

$$x_6 = L \times 4$$

$$= \begin{pmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{pmatrix} \begin{pmatrix} 750 \\ 810 \\ 90 \end{pmatrix} = \begin{pmatrix} 1899 \\ 450 \\ 243 \end{pmatrix}$$

ii, Distribution vectors :-

The characteristic equation is

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$$

$$D_1 = 0$$

$$D_2 = 0 + 0 + (0 - 1.38)$$

$$= -1.38$$

$$D_3 = 0 - 2.3(0) + 0.4(0.18 - 0)$$

$$= 0.4(0.18)$$

$$= 0.072$$

$$\therefore \lambda^3 - 1.38\lambda - 0.072 = 0$$

Eigen values:- $\lambda = 1.2, -1.14, -0.05$

$\lambda = 1.2$ (only +ve value)

Eigen vector :-

$$(A - \lambda I)x = 0$$

Now, $(L - \lambda I)x = 0$

$$\begin{pmatrix} 0-\lambda & 2.3 & 0.4 \\ 0.6 & 0-\lambda & 0 \\ 0 & 0.3 & 0-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1.2 & 2.3 & 0.4 \\ 0.6 & -1.2 & 0 \\ 0 & 0.3 & -1.2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By cross multiplication rules,

$$\begin{array}{cccc} x_1 & x_2 & & x_3 \\ 2.3 & 0.4 & -1.2 & 2.3 \\ -1.2 & 0 & 0.6 & -1.2 \end{array}$$

$$\frac{x_1}{(0+0.48)} = \frac{x_2}{(0.24+0)} = \frac{x_3}{(1.44-1.38)}$$

$$\frac{x_1}{0.48} = \frac{x_2}{0.24} = \frac{x_3}{0.06}$$

$$\frac{x_1}{8} = \frac{x_2}{4} = \frac{x_3}{1}$$

$$\therefore X = \begin{pmatrix} 8 \\ 4 \\ 1 \end{pmatrix}$$

Now,

$$8x + 4x + 1x = 1500$$

$$13x = 1500$$

$$x = \frac{1500}{13}$$

$$\boxed{x = 115.4}$$

In class I,

The no. of females $= 8x$

$= 8(115.4)$

$= 923$

In class II

The no. of females $= 4x$

$= 4(115.4)$

$= 461$

In class III

The no. of females $= 1x$

$= 1(115\cdot 4)$

$= 115$

$\therefore$ The rate of change is $\lambda = 1\cdot 2$

2) What is the number of finals in each class after 3, 6, 9 years if each class initially consist of 400 females. Let leslie matrix be, $L = \begin{bmatrix} 0 & 2\cdot 3 & 0\cdot 4 \\ 0\cdot 6 & -1\cdot 2 & 0 \\ 0 & 0\cdot 3 & -1\cdot 2 \end{bmatrix}$

B) For what initial distribution will number of females in each class change by same proportion. What is the rate of change.

Soln:- Initially $x_0 = \begin{pmatrix} 400 \\ 400 \\ 400 \end{pmatrix}$

After 3 years, the no. of females in each class

$$x_3 = Lx_0 = \begin{bmatrix} 0 & 2\cdot 3 & 0\cdot 4 \\ 0\cdot 6 & -1\cdot 2 & 0 \\ 0 & 0\cdot 3 & -1\cdot 2 \end{bmatrix}_{3\times 3} \begin{pmatrix} 400 \\ 400 \\ 400 \end{pmatrix}_{3\times 1}$$

$$= \begin{pmatrix} 1080 \\ -240 \\ -360 \end{pmatrix}$$

$$x_6 = Lx_0 = \begin{pmatrix} 0 & 2\cdot 3 & 0\cdot 4 \\ 0\cdot 6 & -1\cdot 2 & 0 \\ 0 & 0\cdot 3 & -1\cdot 2 \end{pmatrix} \begin{pmatrix} 1080 \\ -240 \\ -360 \end{pmatrix}$$

$$= \begin{pmatrix} -696 \\ 936 \\ 360 \end{pmatrix}$$

$$x_9 = Lx_0 = \begin{pmatrix} 0 & 2.3 & 0.4 \\ 0.6 & -1.2 & 0 \\ 0 & 0.3 & -1.2 \end{pmatrix} \begin{pmatrix} -696 \\ 936 \\ 360 \end{pmatrix}$$

$$= \begin{pmatrix} 2296.8 \\ -1540.8 \\ -151.2 \end{pmatrix}$$

ii) Distribution vectors

The characteristic equation is

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$$

$$D_1 = -2.4$$

$$D_2 = (1.44 - 0) + 0 + (0 - 1.38)$$

$$= 1.44 - 1.38$$

$$= 0.06$$

$$D_3 = 0(1.44 - 0) - 2.3(-0.72 - 0) + 0.4(0.18 - 0)$$

$$= 0 - 2.3(-0.72) + 0.4(0.18)$$

$$= 1.656 + 0.072$$

$$= 1.728$$

$$\therefore \lambda^3 + 2.4\lambda^2 + 0.06\lambda - 1.728 = 0$$

Eigen values

$$\lambda = 0.73, -1.87, -1.25$$

$$\lambda = 0.73 \text{ (only +ve value)}$$

Eigen vector :-

$$(A - \lambda I)X = 0$$

Now, $(L - \lambda I)X = 0$

$$\begin{pmatrix} 0-\lambda & 2.3 & 0.4 \\ 0.6 & -1.2-\lambda & 0 \\ 0 & 0.3 & -1.2-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -0.73 & 2.3 & 0.4 \\ 0.6 & -1.93 & 0 \\ 0 & 0.3 & -1.93 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By cross multiplication rules,

$$\begin{array}{ccccccc} & x_1 & & x_2 & & x_3 & \\ 2.3 & & 0.4 & & -0.73 & & 2.3 \\ -1.93 & & 0 & & 0.6 & & -1.93 \end{array}$$

$$\frac{x_1}{(0+0.78)} = \frac{x_2}{(0.24+0)} = \frac{x_3}{(1.40-1.38)}$$

$$\frac{x_1}{0.78} = \frac{x_2}{0.24} = \frac{x_3}{0.02}$$

$$\frac{x_1}{39} = \frac{x_2}{12} = \frac{x_3}{1}$$

$$\therefore X = \begin{pmatrix} 39 \\ 12 \\ 1 \end{pmatrix}$$

Now,

$$39x + 12x + 1x = 1200$$

$$52x = 1200$$

$$x = \frac{1200}{52}$$

$$\boxed{x = 23.07}$$

In class I

The no. of females $= 39x$

$= 39(23.07) = 899$

In class II

The no. of females $= 12x$

$= 12(23.07)$

$= 276$

In class III

The no. of females $= 1 \cdot x$

$= 1(23.07)$

$= 23$

$\therefore$ The rate of change is

$\lambda = 0.73$