23MAT101 26/09/2023

Unit - 1 Period :- 2

Matrices and calculus

Characteristic equation :-

The equation determinant $|A - \lambda I| = 0$ is called characteristic equation.

Here A is the square matrix of order n.

$\lambda$ is the scalar

I is the identity matrix of order n.

Note :-

1, For a 2x2 matrix, the characteristic equation is $\lambda^2 - D_1\lambda + D_2 = 0$

Here, $D_1$ = sum of the main diagonal elements.

$D_2 = |A|$

2, For a 3x3, the characteristic equation is $\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$

Here, $D_1$ = sum of the main diagonal elements.

$D_2$ = sum of the minors of the main diagonal element

$D_3 = |A|$

Q// Find the characteristic equation of matrix.

i, $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

The characteristic equation is $\lambda^2 - D_1\lambda + D_2 = 0$

Here, $D_1$ = sum of the main diagonal elements.

$= 1 + 2 = 3$

$D_2 = |A| = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

$\therefore \lambda^2 - 3\lambda + 2 = 0$

ii) $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

let $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

$D_1 = 1 + 3 = 4$

$D_2 = |A| = \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} = 3 - 8 = -5$

$\therefore \lambda^2 - 4\lambda - 5 = 0$

iii) $\begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$

let $A = \begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$

The characteristic equation is

$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$

Here,

$D_1$ = sum of the main diagonal element.

$= 2 + 1 - 4 = -1$

$D_2$ = Sum of the minors of the main diagonal elements.

$D_2 = \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ -5 & -4 \end{vmatrix} + \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix}$

$= (-4 - 6) + (-8 + 5) + (2 + 9)$

$= -10 - 3 + 11$

$D_2 = -2$

$D_3 = |A| = \begin{vmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{vmatrix}$

$= 2\begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} - (-3)\begin{vmatrix} 3 & 3 \\ -5 & -4 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ -5 & 2 \end{vmatrix}$

$= 2(-4-6) + 3(-12+15) + 1(6+5)$

$= 2(-10) + 3(3) + 1(11)$

$= -20 + 9 + 11 = 0$

$\lambda^3 + \lambda^2 - 2\lambda - 0 = 0$

iv) $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

$D_1 = 8 + 7 + 3 = 18$

$D_2 = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$

$= (21-16) + (24-4) + (56-36)$

$= 5 + 20 + 20 = 45$

$D_3 = 8\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} - (-6)\begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} -6 & 7 \\ 2 & -4 \end{vmatrix}$

$= 8(21-16) + 6(-18+8) + 2(24-14)$

$= 8(5) + 6(-10) + 2(10)$

$= 40 - 60 + 20 = 0$

$\lambda^3 - 18\lambda^2 + 45\lambda - 0 = 0$