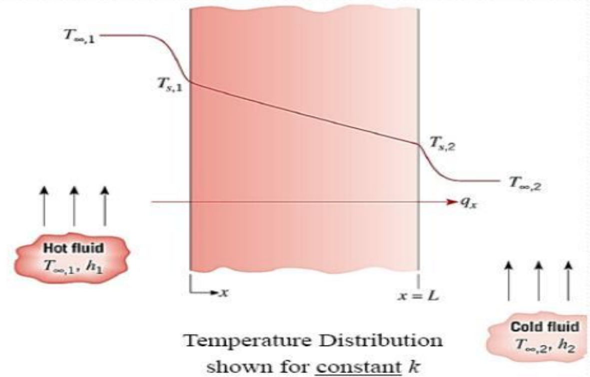
Chapter 3 : One-dimensional, Steady state conduction (without thermal generation)

3.2 The plane wall – temperature distribution



Temperature Distribution shown for constant k

Boundary conditions: $T(0) = T_{s,1}$

$T(L) = T_{s,2}$

Using Eq. (2.2) in Chapter 2, by assuming steady-state conditions and no internal heat generation (i.e. $q = 0$), then the 1-D heat conduction equation reduces to:

$$\frac{d}{dx}\left(k \cdot A \frac{dT}{dx}\right) = 0$$

For constant k and A, second order differential equation:

$$\frac{d^2T}{dx^2} = 0$$

This mean:

Heat flux (q''_x) is independent of x

Heat rate (q_x) is independent of x

4

Two Dimensional Heat conduction

Generalized GDE: $$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + q''' = \rho c_p\left(\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} + W\frac{\partial T}{\partial z}\right)$$

1D steady: GDE is an Ordinary Differential Equation (ODE) $\quad \frac{\partial^2 T}{\partial x^2} = 0$ or $\frac{d^2 T}{dx^2} = 0$

2D/3D steady: GDE is a Partial Differential Equation (PDE) $\quad \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$

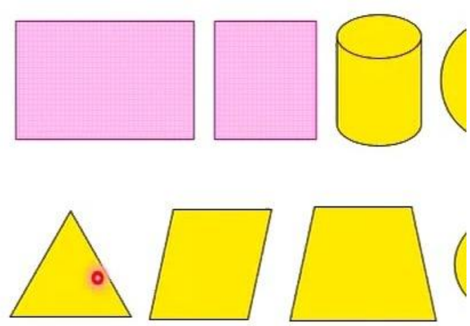
1D unsteady: GDE is a partial Differential Equation (PDE) $\rightarrow \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

Unlike ODE, no general solution method is available for PDE

How to solve a PDE?

- Separation of variables
- Laplace transform

Multi-dimensional conduction problem



When the boundaries correspond to the coordinate surfaces in a system of orthogonal coordinates (e.g. Cartesian, cylindrical, spherical coordinates), an exact solution is possible