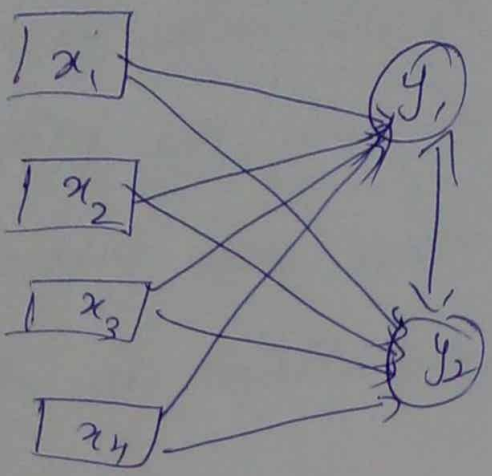
Self organizing maps.



→ Consider the nlw shown in figure which considers four training samples each vector of length 4 and 2 output units.

→ Train the SOFM (Self organizing feature Map) network by determining the class memberships of the input data.

$$x_1 = (1, 0, 1, 0)$$

$$x_2 = (1, 0, 0, 0)$$

$$x_3 = (1, 1, 1, 1)$$

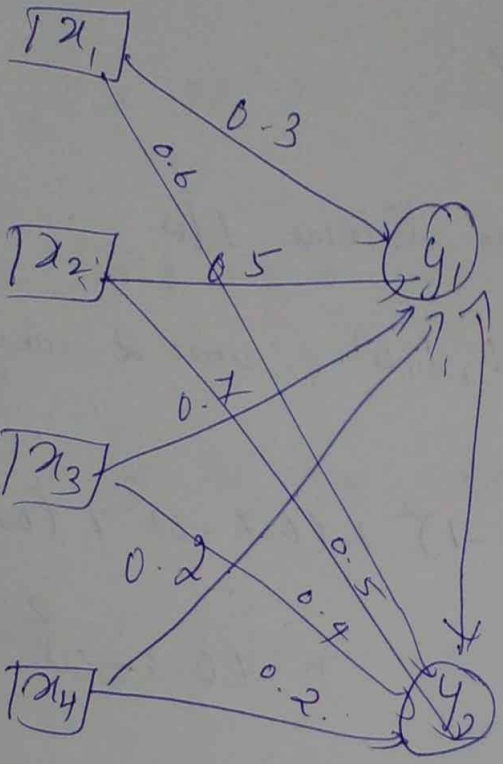
$$x_4 = (0, 1, 1, 0)$$

unit 1, unit 2 = O/P units.

learning rate (α) 0.6.
η(t).

Initial weight matrix.

$$\begin{matrix} \text{unit 1} \\ \text{unit 2} \end{matrix} \begin{bmatrix} 0.3 & 0.5 & 0.7 & 0.2 \\ 0.6 & 0.5 & 0.4 & 0.2 \end{bmatrix}$$



Iteration 1:

Training Sample x_1 : (1, 0, 1, 0)

Weight matrix:

$$\begin{bmatrix} \text{unit 1} \\ \text{unit 2} \end{bmatrix} = \begin{bmatrix} 0.3 & 0.5 & 0.7 & 0.2 \\ 0.6 & 0.7 & 0.4 & 0.3 \end{bmatrix}$$

Compute Euclidean distance b/w

$x_i : (1, 0, 1, 0)$ & unit 1 weights.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d^2 = (0.3 - 1)^2 + (0.5 - 0)^2 + (0.7 - 1)^2 + (0.2 - 0)$$

$$= 0.87$$

Compute Euclidean distance b/w

$x_i : (1, 0, 1, 0)$ & unit 2 weights

$$d^2 = (0.6 - 1)^2 + (0.7 - 0)^2 + (0.4 - 1)^2 + (0.3 - 0)^2$$

$$= 1.1$$

compare unit 1 & unit 2 weight

unit 1 is low distance.

So unit 1 weight is wins.

Update unit 1 weight

$$w_j(t+1) = w_j(t) + \eta(t)(x_s - w_j(t))$$

new weight — new weight

Update the weights of the winning unit

New unit 1 weights:

$$= [0.3, 0.5, 0.7, 0.2] + 0.6[1\ 0\ 1\ 0] -$$

old unit 1 weight $[0.3\ 0.5\ 0.7\ 0.2]$

$$= [0.3\ \ 0.5\ \ 0.7\ \ 0.2] + 0.6[0.7\ \ -0.5\ \ 0.3\ \ -02]$$

$$= [0.3\ \ 0.5\ \ 0.7\ \ 0.2] + [0.42\ \ -0.30\ \ 0.18\ \ -0.12]$$

$$= [0.72\ \ 0.2\ \ 0.88\ \ 0.08]$$

$$\begin{bmatrix} \text{unit 1} \\ \text{unit 2} \end{bmatrix} = \begin{bmatrix} 0.72 & 0.2 & 0.88 & 0.8 \\ 0.6 & 0.7 & 0.4 & 0.3 \end{bmatrix}$$

Iteration 2 sample 2.

Training sample x_2: (1, 0, 0, 0)

Weight matrix:

$$\begin{bmatrix} \text{unit 1} \\ \text{unit 2} \end{bmatrix} = \begin{bmatrix} 0.72 & 0.2 & 0.88 & 0.8 \\ 0.6 & 0.7 & 0.4 & 0.3 \end{bmatrix}$$

compute Euclidean distance between $x_1(1,0,0,0)$ and unit 1 weights.

$$d^2 = (0.72-1)^2 + (0.2-0)^2 + (0.88-0)^2 + (0.08-0)^2$$

$$= 0.74.$$

compute Euclidean distance b/w $x_2:(1,0,0,0)$ and unit 2 weights.

$$d^2 = (0.6-1)^2 + (0.7-0)^2 + (0.4-0)^2 + (0.3-0)^2$$

$$= 0.9$$

unit 1 wins

update unit 1 weight

$$w_j(t+1) = w_j(t) + \eta(t)(x_s - w_j(t))$$

update the weights of the winning unit.

New unit 1 weight $= [0.72\ 0.2\ 0.88\ 0.08] + 0.6$

$$(1\ 0\ 0\ 0) - (0.72\ 0.2\ 0.88\ 0.08)$$

$$= [0.72 \quad 0.2 \quad 0.88 \quad 0.08] + 0.6[0.28 \quad -0.2 \quad -0.88 \quad -0.08]$$

$$= [0.72 \quad 0.2 \quad 0.88 \quad 0.08] + [0.17 \quad -0.12 \quad -0.53 \quad -0.05]$$

$$= [0.89 \quad 0.08 \quad 0.35 \quad 0.03]$$

$$\begin{bmatrix} \text{unit } 1 \\ \text{unit } 2 \end{bmatrix} = \begin{bmatrix} 0.89 & 0.08 & 0.35 & 0.03 \\ 0.6 & 0.7 & 0.4 & 0.3 \end{bmatrix}$$

Iteration 3:

Training sample x_3 : (1, 1, 1, 1).

weight matrix:

$$\begin{bmatrix} \text{unit } 1 \\ \text{unit } 2 \end{bmatrix} = \begin{bmatrix} 0.89 & 0.08 & 0.35 & 0.03 \\ 0.6 & 0.7 & 0.4 & 0.3 \end{bmatrix}$$

Compute Euclidean distance blw x_3 : (1, 1, 1, 1) and unit 1 weights.

$$d^2 = (0.89-1)^2 + (0.08-1)^2 + (0.35-1)^2 + (0.03-1)^2$$

$$= 2.2.$$

compute Euclidean distance between $x_3 : (1, 1, 1, 1)$ and unit 2 weights

$$d^2 = (0.6-1)^2 + (0.7-1)^2 + (0.4-1)^2 + (0.3-1)^2$$

$$= 1.1$$

unit 2 wins.

update unit 2 weights.

$$w_j(t+1) = w_j(t) + \eta(t)(x_s - w_j(t))$$

~~Update unit 2 weights of the win~~

Update the weights of the winning unit.

New unit 2 weights $\Rightarrow$

$$[0.6 \quad 0.7 \quad 0.4 \quad 0.3] + 0.6[1, 1 1 1] -$$

$$[0.6 \quad 0.7 \quad 0.4 \quad 0.3]$$

$$\Rightarrow [0.6 \quad 0.7 \quad 0.4 \quad 0.3] + 0.6 [0.4 \quad 0.3 \quad 0.6 \quad 0.7]$$

$$= [0.6 \quad 0.7 \quad 0.4 \quad 0.3] + [0.24 \quad 0.18 \quad 0.36 \quad 0.42]$$

$$= [0.84 \quad 0.88 \quad 076 \quad 0.72]$$

$$\begin{bmatrix} \text{unit 1} \\ \text{unit 2} \end{bmatrix} = \begin{bmatrix} 0.89 & 0.08 & 0.35 & 0.03 \\ 0.84 & 0.88 & 0.76 & 0.72 \end{bmatrix}$$

Iteration 4.

Training sample $x_4 = (0, 1, 1, 0)$

weight matrix:

$$\begin{bmatrix} \text{unit 1} \\ \text{unit 2} \end{bmatrix} = \begin{bmatrix} 0.89 & 0.08 & 0.35 & 0.03 \\ 0.84 & 0.88 & 0.76 & 0.72 \end{bmatrix}$$

compute Euclidean distance between $x_4 : (0, 1, 1, 0)$ and unit 1 weights.

$$d_1^2 = (0.89 - 0)^2 + (0.08 - 1)^2 + (0.35 - 1)^2 + (0.03 - 0)^2$$

$$= 2.06.$$

compute Euclidean distance between $x_4 : (0, 1, 1, 0)$ and unit 2 weights

$$d_2^2 = (0.84 - 0)^2 + (0.88 - 1)^2 + (0.76 - 1)^2 + (0.72 - 0)^2$$

$$d_2^2 = 1.3.$$

unit 2 wins.

update unit 2 weights

$$w_j(t+1) = w_j(t) + \eta(t)(x_s - w_j(t))$$

new unit 2 weights $= [0.84 \ 0.88 \ 0.76 \ 0.72] + 0.6($

$[0 \ 1 \ 1 \ 0] - [0.84 \ 0.88 \ 0.76 \ 0.72])$

$= [0.84 \ 0.88 \ 0.76 \ 0.72] + 0.6[-0.84 \ 0.12 \ 0.24 \ -0.72]$

$= [0.84 \ 0.88 \ 0.76 \ 0.72] + [-0.5 \ 0.07 \ 0.14 \ -0.43]$

$= [0.34 \ 0.95 \ 0.9 \ 0.29]$

$$\begin{bmatrix} \text{unit } 1 \\ \text{unit } 2 \end{bmatrix} = \begin{bmatrix} 0.89 & 0.08 & 0.35 & 0.03 \\ 0.34 & 0.95 & 0.9 & 0.29 \end{bmatrix}$$

Best mapping units for each of the sample taken are;

x_1 : (1, 0, 1, 0) → unit 1

x_2 : (1, 0, 0, 0) → unit 1

x_3 : (1, 1, 1, 1) → unit 2

x_4 : (0, 1, 1, 0) → unit 2

} Epoch one.

This prouss is continued for many epochs until the feature map does not change.