

$$\frac{Y(z)}{z} = \frac{2}{z-1/4} - \frac{1}{z-1/2}$$

$$Y(z) = 2\left(\frac{z}{z-1/4}\right) - \frac{z}{z-1/2}$$

$$Y(n) = 2\left(\frac{1}{4}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(n)$$

Convolution Sum

Convolution of sum is defined as

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

Four steps involved in computing convolution:-

- ① Folding
- ② Shifting
- ③ Multiplication
- ④ Summation.

Four methods available in compute convolution sum:

- ① Definition method
- ② Graphical
- ③ Tabulation
- ④ Multiplication.

Let M be the total of samples of $x(n)$ and N be the total no. of samples of $h(n)$ then the total no of samples in $y(n)$ be the $\boxed{M+N-1}$.

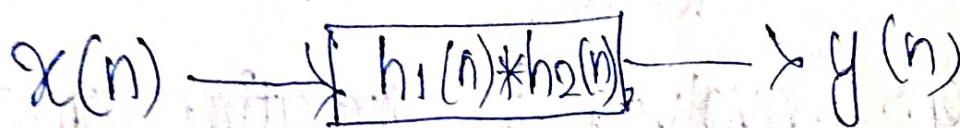
Properties of convolution sum

① Commutative property

$$y(n) = x(n) * h(n) = h(n) * x(n)$$

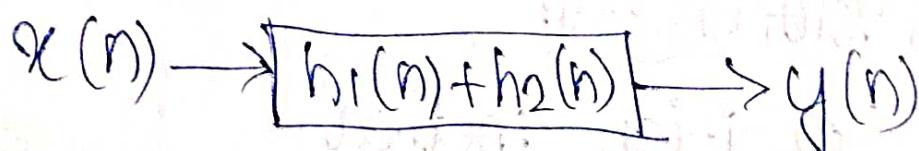
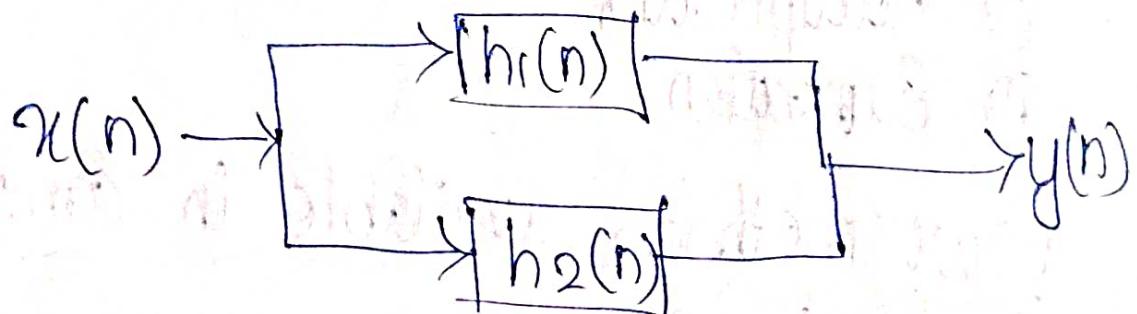
② Associative property

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$



③ Distributive Property

$$x(n) * h_1(n) + x(n) * h_2(n) = x(n) * [h_1(n) + h_2(n)]$$



Q) Find the convolution sum of given sequence

$$x(n) = \{1, 2, 3, 4, 5\}$$

$$h(n) = \{6, 7, 8\}$$

Sol:

$$\begin{array}{cccccc}
 & 1 & 2 & 3 & 4 & 5 \\
 \hline
 & 6 & 7 & 8 & & \\
 \hline
 8 & 16 & 24 & 32 & 40 & \\
 7 & 14 & 21 & 28 & 35 & \\
 6 & 12 & 18 & 24 & 30 & \\
 \hline
 6 & 19 & 40 & 61 & 82 & 67 & 40 \\
 \hline
 g(n) = \{6, 19, 40, 61, 82, 67, 40\}
 \end{array}$$

Tabulation Method

	6	7	8
1	6	7	8
2	12	14	16
3	18	21	24
4	24	28	32
5	30	35	40

$$g(n) = \{6, 19, 40, 61, 82, 67, 40\}$$

Definition Method

$$g(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

$$x(0) = 1 \quad h(0) = 6$$

$$x(1) = 2 \quad h(1) = 7$$

$$x(2) = 3 \quad h(2) = 8$$

$$x(3) = 4$$

$$x(4) = 5$$

$$N=5$$

$$N=3$$

Total no of samples : $M+N-1$
 $= 5 + 3 - 1$

$$y(n) = \sum_{k=0}^4 x(k) h(n-k)$$

$n=0$

$$\begin{aligned} y(0) &= \sum_{k=0}^4 x(k) h(-k) \\ &= x(0) h(0) + x(1) h(-1) + x(2) h(-2) + x(3) \\ &\quad + x(4) h(-3) + x(5) \\ &= 6 \end{aligned}$$

$n=1$

$$\begin{aligned} y(1) &= \sum_{k=0}^4 x(k) h(1-k) \\ &= x(0) h(1) + x(1) h(0) + x(2) h(-1) + x(3) h(-2) \\ &\quad + x(4) h(-3) \\ &= (1 \times 7) + (2 \times 6) \\ &= 19 \end{aligned}$$

$n=2$

$$\begin{aligned} y(2) &= \sum_{k=0}^4 x(k) h(2-k) \\ &= x(0) h(0) + x(1) h(1) + x(2) h(0) + x(3) h(-1) \\ &\quad + x(4) h(-2) \end{aligned}$$

$$= (1 \times 8) + (2 \times 7) + (8 \times 6)$$

$$= 8 + 14 + 48$$

$$= 40$$

$$\underline{n=3} \quad y(3) = \sum_{k=0}^4 x(k) h(3-k)$$

$$y(3) = \sum_{k=0}^4 x(k) h(3-k)$$

$$= x(0) h(3) + x(1) h(2) + x(2) h(1) + x(3) h(0)$$

$$= 16 + 21 + 24$$

$$= 61$$

$$\underline{n=4}$$

$$y(4) = \sum_{k=0}^4 x(k) h(4-k)$$

$$= x(0) h(4) + x(1) h(3) + x(2) h(2) + x(3) h(1) + x(4) h(0)$$

$$= 24 + 28 + 30$$

$$= 82$$

$$\underline{n=5}$$

$$y(5) = \sum_{k=0}^4 x(k) h(5-k)$$

$$= x(0) h(5) + x(1) h(4) + x(2) h(3) + x(3) h(2) + x(4) h(1)$$

$$= 32 + 35$$

$$= 67$$

$$\begin{aligned}
 n &= 6 \\
 y(6) &= \sum_{k=0}^4 x(k) h(6-k) \\
 &= x(0) h(6) + x(1) h(5) + x(2) h(4) + \\
 &\quad x(3) h(3) + x(4) h(2) \\
 &= 40
 \end{aligned}$$

~~$(0.125) + (0.250) + (0.375) +$~~

~~$(0.400) + (0.425)$~~

~~$(0.450) + (0.475)$~~

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~~$(1.200) + (1.225)$~~

~~$(1.250) + (1.275)$~~

~~$(1.300) + (1.325)$~~

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