

$$Y(z) = \frac{2}{z-1/4} - \frac{1}{z-1/2}$$

$$Y(z) = 2 \left(\frac{z}{z-1/4} \right) - \frac{z}{z-1/2}$$

$$Y(n) = 2 \left[(1/4)^n u(n) - (-1/2)^n u(n) \right]$$

Convolution Sum

Convolution of sum is defined as

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

Four steps involved in computing convolution:

- ① Folding
- ② Shifting
- ③ Multiplication
- ④ Summation

Four methods available in compute convolution sum:

- ① Definition method
- ② Graphical
- ③ Tabulation
- ④ Multiplication

Let M be the total of samples of $x(n)$ and N be the total no. of samples of $h(n)$ then the total no of samples in $y(n)$ be the $M+N-1$.

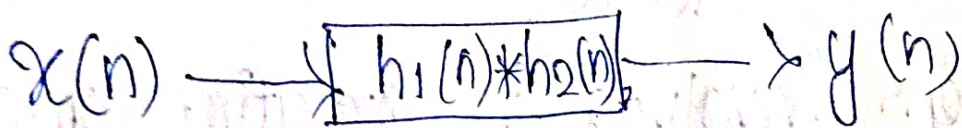
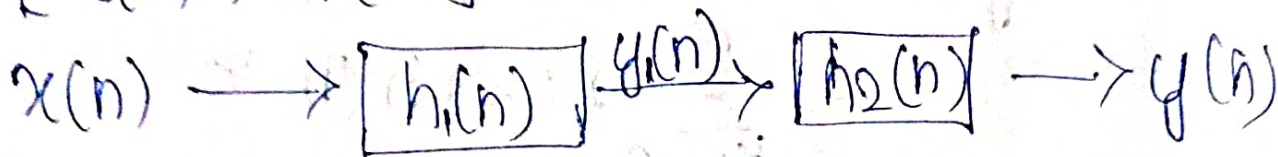
Properties of convolution sum

① Commutative property

$$y(n) = x(n) * h(n) = h(n) * x(n)$$

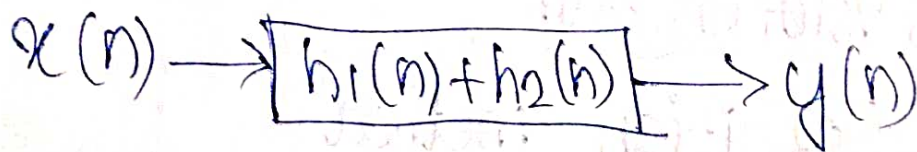
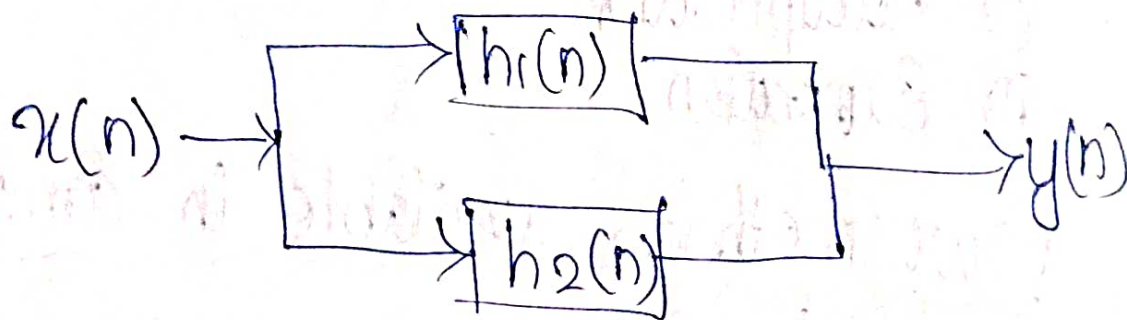
② Associative property

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$



③ Distributive Property

$$x(n) * h_1(n) + x(n) * h_2(n) = x(n) * [h_1(n) + h_2(n)]$$



Q) Find the convolution sum of given sequence
 $x(n) = \{1, 2, 3, 4, 5\}$
 $h(n) = \{6, 7, 8\}$

Sol:

| | | | | | | |
|---|----|----|----|----|----|----|
| | 1 | 2 | 3 | 4 | 5 | |
| | | | 6 | 7 | 8 | |
| | | 8 | 16 | 24 | 32 | 40 |
| 7 | 14 | 21 | 28 | 35 | | |
| 6 | 12 | 18 | 24 | 30 | | |
| 6 | 19 | 40 | 61 | 82 | 67 | 40 |

$$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$$

Tabulation Method

| | | | |
|---|----|----|----|
| | 6 | 7 | 8 |
| 1 | 6 | 7 | 8 |
| 2 | 12 | 14 | 16 |
| 3 | 18 | 21 | 24 |
| 4 | 24 | 28 | 32 |
| 5 | 30 | 35 | 40 |

$$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$$

Definition Method

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

$$x(0) = 1 \quad h(0) = 6$$

$$x(1) = 2 \quad h(1) = 7$$

$$x(2) = 3 \quad h(2) = 8$$

$$x(3) = 4$$

$$x(4) = 5$$

$$M = 5$$

$$N = 3$$

$$\text{Total no of samples : } M + N - 1$$

$$= 5 + 3 - 1$$

$$y(n) = 7$$

$$y(n) = \sum_{k=0}^4 x(k) h(n-k)$$

$$\underline{n=0}$$

$$y(0) = \sum_{k=0}^4 x(k) h(-k)$$

$$= x(0)h(0) + x(1)h(-1) + x(2)h(-2) + x(3)h(-3) + x(4)h(-4)$$

$$= 6$$

$$\underline{n=1}$$

$$y(1) = \sum_{k=0}^4 x(k) h(1-k)$$

$$= x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2) + x(4)h(-3)$$

$$= (1 \times 7) + (2 \times 6)$$

$$= 19$$

$$\underline{n=2}$$

$$y(2) = \sum_{k=0}^4 x(k) h(2-k)$$

$$= x(0)h(2) + x(1)h(1) + x(2)h(0) + x(3)h(-1) + x(4)h(-2)$$

$$= (1 \times 8) + (2 \times 7) + (3 \times 6)$$

$$= 8 + 14 + 18$$

$$= 40$$

$$\underline{n=3}$$

$$y(3) = \sum_{k=0}^4 x(k) h(3-k)$$

$$= x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0)$$

$$= 16 + 21 + 24$$

$$= 61$$

$$\underline{n=4}$$

$$y(4) = \sum_{k=0}^4 x(k) h(4-k)$$

$$= x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) + x(4)h(0)$$

$$= 24 + 28 + 30$$

$$= 82$$

$$\underline{n=5}$$

$$y(5) = \sum_{k=0}^4 x(k) h(5-k)$$

$$= x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) + x(4)h(1)$$

$$= 32 + 35$$

$$= 67$$

$$\underline{n=6}$$

$$y(6) = \sum_{k=0}^H x(k) h(6-k)$$
$$= x(0)h(6) + x(1)h(5) + x(2)h(4) +$$
$$x(3)h(3) + x(4)h(2)$$
$$= 40$$

$$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$$