Q) Find the output of the system whose input output relate by

$$y(n) = 7y(n-1) - 12y(n-2) + 2x(n) - x(n-2)$$

For input $x(n) = u(n)$.

Soln:

$$y(n) = 7y(n-1) - 12y(n-2) + 2x(n) - x(n-2)$$

$$y(z) = 7z^{-1}y(z) - 12z^{-2}y(z) + 2x(z) - z^{-2}x(z)$$

$$y(z) - 7z^{-1}y(z) + 12z^{-2}y(z) = 2x(z) - z^{-2}x(z)$$

$$y(z)[1 - 7z^{-1} + 12z^{-2}] = x(z) = [2 - z^{-2}]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - z^{-2}}{1 - 7z^{-1} + 12z^{-2}}$$

Multiply & divide by z^2

$$y(z) = \frac{z^2}{z^2} = \frac{2 - z^{-2}}{1 - 7z^{-2} + 12z^{-2}}$$

$$y(z) = \frac{2z^2 - 1}{z^2 - 7z + 12}$$

$$x(n) = u(n) = \frac{z}{z-1}$$

$$y(z) = \frac{z(2z^2 - 1)}{(z-1)(z^2 - 7z + 12)}$$

$$\frac{y(z)}{z} = \frac{2z^2-1}{(z-1)(z^2-7x+12)}$$

$$\frac{y(z)}{z} = \frac{2z^2-1}{(z-1)(z-3)(z-4)}$$

$$\frac{2z^2-1}{(z-1)(z-3)(z-4)} = \frac{A}{(z-1)} + \frac{B}{(z-3)} + \frac{C}{(z-4)}$$

$$2z^2-1 = A(z-3)(z-4) + B(z-1)(z-4) + C(z-1)(z-3)$$

Put $z=1$,

$$1 = A(-2)(-3) + B(0) + C(0)$$

$$\boxed{A = 1/6}$$

Put $z=3$

$$17 = A(0) + B(2)(-1) + C(0)$$

$$\boxed{B = -17/2}$$

Put $z=4$

$$31 = A(0) + B(0) + C(3)(1)$$

$$\boxed{C = 31/3}$$

$$\frac{y(z)}{z} = \frac{1}{6}\left(\frac{1}{z-1}\right) - \frac{17}{2}\left(\frac{1}{z-3}\right) + \frac{31}{3}\left(\frac{1}{z-4}\right)$$

$$y(z) = \frac{1}{6}\left(\frac{z}{z-1}\right) - \frac{17}{2}\left(\frac{z}{z-3}\right) + \frac{31}{3}\left(\frac{z}{z-4}\right)$$

$$y(n) = \frac{1}{6}u(n) - \frac{17}{2}(3)^n u(n) + \frac{31}{3}(4)^n u(n)$$

Q) The output $y(n)$ of a discrete time system LTI is $2(1/3)^n u(n)$, when $x(n) = u(n)$. Find the impulse response.

$$y(z) = 2\left(\frac{1}{3}\right)^n u(n)$$

$$= 2\left(\frac{Z}{Z-1/3}\right)$$

$$x(z) = \frac{Z}{Z-1}$$

$$H(z) = \frac{y(z)}{x(z)}$$

$$= \frac{2\left(\frac{Z}{Z-1/3}\right)}{\frac{Z}{Z-1}}$$

$$= \frac{2(Z-1)}{Z-1/3}$$

Multiply & divide by z,

$$\frac{H(Z)}{Z} = \frac{2(Z-1)}{Z(Z-1/3)}$$

$$\frac{2(Z-1)}{Z(Z-1/3)} = \frac{A}{Z} + \frac{B}{Z-1/3}$$

$$2(Z-1) = A(Z-1/3) + B(Z)$$

Put $z=0$,

$$-2 = A(-1/3) + B(0)$$

$$\boxed{A=6}$$

Put $Z = 1/3$

$$-\frac{4}{3} = A(0) + B(1/3)$$

$$\boxed{B = -4}$$

$$\frac{H(Z)}{Z} = \frac{6}{Z} - \frac{4}{Z - 1/3}$$

$$H(Z) = 6\frac{Z}{Z} - 4\left(\frac{Z}{Z - 1/3}\right)$$

$$h(n) = 6\,\delta(n) - 4\,(1/3)^n u(n)$$

Q) A discrete time causal system has a function $H(Z) = \frac{1 - Z^{-1}}{1 - 0.2Z^{-1} - 0.15Z^{-2}}$.

Determine difference equation, Impulse response, pole zero diagram.

Sol:

$$H(Z) = \frac{1 - Z^{-1}}{1 - 0.2Z^{-1} - 0.15Z^{-2}}$$

$$H(Z) = \frac{Y(Z)}{X(Z)}$$

$$Y(Z)\left[1 - 0.2Z^{-1} - 0.15Z^{-2}\right] = X(Z)\left[1 - Z^{-1}\right]$$

$$y(Z) - 0.2Z^{-1}y(Z) - 0.15Z^{-2} = X(Z) - Z^{-1}X(Z)$$

$$y(n) - 0.2y(n-1) - 0.15y(n-2) = x(n) - x(n-1)$$

Impulse Response,

$$H(z) = \frac{1 - z^{-1}}{1 - 0.2z^{-1} - 0.15z^{-2}}$$

Multiply & divide by z^2,

$$H(z) = \frac{z^2}{z^2} \cdot \frac{1 - z^{-1}}{1 - 0.2z^{-1} - 0.15z^{-2}}$$

$$H(z) = \frac{z^2 - z}{z^2 - 0.2z - 0.15}$$

$$\frac{H(z)}{z} = \frac{z-1}{z^2 - 0.2z - 0.15}$$

$$\frac{H(z)}{z} = \frac{z-1}{(z+0.3)(z-0.5)}$$

$$\frac{z-1}{(z+0.3)(z-0.5)} = \frac{A}{z+0.3} + \frac{B}{z-0.5}$$

$$z-1 = A(z-0.5) + B(z+0.3)$$

Put $z = -0.3$

$$-1.3 = A(-0.8) + B(0)$$

$$A = \frac{1.3}{0.8}$$

$$\boxed{A = 1.625}$$

Put $Z = 0.5$

$$-0.5 = A(0) + B(0.8)$$

$$\boxed{B = 0.625}$$

$$\frac{H(Z)}{Z} = \frac{1.625}{Z+0.3} + \frac{0.625}{Z-0.5}$$

$$H(Z) = 1.625\left(\frac{Z}{Z+0.3}\right) + 0.625\left(\frac{Z}{Z-0.5}\right)$$

$$h(n) = 1.625\,(-0.3)^n u(n) + 0.625(0.5)^n u(n)$$

Poles,

$Z = 0$

$P_1 = -0.3$

$P_2 = 0.5$

Q) A discrete time LTI system is defined by $y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n)$ Determine system transfer function, Impulse response & Frequency response.

Sol: $y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n)$

Taking Z-transform.

$$y(Z) - \frac{3}{4} Z^{-1} y(Z) + \frac{1}{8} y(Z) = x(Z)$$

$$y(Z)\left[1 - \frac{3}{4} Z^{-1} + \frac{1}{8} Z^{-2}\right] = x(Z)$$

$$H(Z) = \frac{y(Z)}{x(Z)} = \frac{1}{1 - \frac{3}{4}Z^{-1} + \frac{1}{8}Z^{-2}}$$

For frequency response,

$$H(e^{j\omega}) = \frac{1}{1 - \frac{3}{4}(e^{j\omega})^{-1} + \frac{1}{8}(e^{j\omega})^{-2}}$$

Impulse Response,

Multiply and Divide by Z^2

$$H(Z) = \frac{Z^2}{Z^2} \cdot \frac{1}{1 - \frac{3}{4}Z^{-1} + \frac{1}{8}Z^{-2}}$$

$$= \frac{Z^2}{Z^2 - \frac{3}{4}Z + \frac{1}{8}}$$

$$\frac{y(Z)}{Z} = \frac{Z}{(Z-\frac{1}{2})(Z-\frac{1}{4})}$$

$$\frac{Z}{(Z-\frac{1}{2})(Z-\frac{1}{4})} = \frac{A}{(Z-\frac{1}{2})} + \frac{B}{(Z-1/4)}$$

$$Z = A(Z-1/4) + B(Z-1/2)$$

Put $Z = 1/4$

$$\frac{1}{4} = A(0) + B\left(\frac{-1}{4}\right)$$

$$\boxed{B = \frac{-1}{4} \times 4} \quad \boxed{B = -1}$$

Put $Z = 1/2$

$$1/2 = (1/4)A + B(0) \quad \boxed{A = 2}$$