UNIT-V

Linear Time Invarient - Discrete Time System

Difference Equation

It is an efficient way to implement discrete time system.

It is defined as the convolution of input sequence $x(n)$ and unit sample response $h(n)$ gives output $y(n)$

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K)$$

* Finite Impulse Response System [FIR]

The system for which unit sample response $h(n)$ (or) Impulse response $h(n)$ has finite no. of terms are called FIR

$$y(n) = \sum_{K=0}^{M-1} h(K)\, x(n-K)$$

* Infinite Impulse Response System [IIR]

The system is said to be an infinite impulse response system, if the length of the impulse response is infinite.

$$y(n) = \sum_{K=0}^{\infty} h(K)\, x(n-K)$$

Non-Recursive System

When the output $y(n)$ of the system depends upon the present & past input, then it is called non-recursive system.

$$y(n) = \sum_{K=0}^{M} h(K)\,x(n-K)$$

Recursive System

When the output $y(n)$ of the system depends on present and past inputs as well as past output is called recursive system.

$$y(n) = \sum_{K=0}^{M} x(K)$$

Condition of Causality LTI of Discrete Time system.

The LTI system is causal if & only if

$$h(n) = 0 \text{ for } n < 0$$

Condition for Stability

$$\sum_{K=-\infty}^{\infty} |h(K)| < \infty$$

Z-Transform Analysis

$$H(z) = \frac{Y(z)}{X(z)}$$

$H(z) \rightarrow$ System function.

Q) A difference eqn of system is defined by $y(n) = 0.5\, y(n-1) + x(n)$. Determine

$0.5z^{-1} y(z) + x(z)$

i) System function

ii) Pole zero plot of system function

iii) Unit sample response of the system

Sol:

(i) Z-transform on both sides.

$$y(z) = 0.5z^{-1} y(z) + x(z)$$

$$y(z)[1 - 0.5z^{-1}] = x(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{1}{1 - 0.5z^{-1}}$$

(ii) Multiply & divide by z,

$$H(z) = \frac{z}{z} \cdot \frac{1}{1 - 0.5z^{-1}}$$

$$= \frac{z}{z - 0.5}$$

$Z_1 = 0$

$P_1 = 0.5$

(iii) $h(n) = (0.5)^n u(n)$

Solving difference equation using Z-transform

$$y(n-1) \xleftrightarrow{Z.T} Z^{-1} y(z) + Z^0 y(-1)$$

$$y(n-2) \xleftrightarrow{Z.T} Z^{-2} y(z) + Z^{-1} y(-1) + Z^0 y(-2)$$

$$y(n-3) \xleftrightarrow{Z.T} Z^{-3} y(z) + Z^{-2} y(-1) + Z^{-1} y(-2) + Z^0 y(-3)$$

Q) Given that $y(-1) = 5$, $y(-2) = 0$. Solve the difference equation $y(n) - 3y(n-1) - 4y(n-2) = 0$

Sol:

$$y(z) - 3[Z^{-1} y(z) + Z^0 y(-1)] - 4[Z^{-2} y(z) + Z^{-1} y(-1) + Z^0 y(-2)] = 0$$

$$y(z) - 3[Z^{-1} y(z) + Z^0 (5)] - 4[Z^{-2} y(z) + Z^{-1}(5) + Z^0 (0)] = 0$$

$$y(z) - 3[Z^{-1} y(z) + 5Z^0] - 4[Z^{-2} y(z) + 5Z^{-1}] = 0$$

$$y(z) - 3z^{-1} y(z) - 15z^0 - 4z^{-2} y(z) - 20z^{-1} = 0$$

$$y(z)[1 - 3z^{-1} - 4z^{-2}] = 15z^0 + 20z^{-1}$$

$$y(z) = \frac{15z^0 + 20z^{-1}}{1 - 3z^{-1} - 4z^{-2}}$$

Multiply & divide by z^2

$$y(z) = \frac{z^2}{z^2} \cdot \frac{15z^0 + 20z^{-1}}{1 - 3z^{-1} - 4z^{-2}}$$

$$y(z) = \frac{15z^2+20z}{z^2-3z-4}$$

$$\frac{y(z)}{z} = \frac{15z+20}{z^2-3z-4}$$

$$\frac{y(z)}{z} = \frac{15z+20}{z^2-3z-4} = \frac{15z+20}{(z-4)(z+1)}$$

$$= \frac{A}{z-4} + \frac{B}{z+1}$$

$$15z+20 = A(z+1) + B(z-4)$$

Put $z=-1$

$$5 = A(0) + B(-5)$$

$$\boxed{B=-1}$$

Put $z=4$

$$80 = A(5) + B(0)$$

$$\boxed{A=16}$$

$$\frac{H(z)}{z} = \frac{16}{z-4} - \frac{1}{z+1}$$

$$H(z) = 16\frac{z}{z-4} - \frac{z}{z+1}$$

$$h(n) = 16(4)^n u(n) - (-1)^n u(n)$$