Inverse Z-Transform

(1) Partial fraction method
(2) Power series expansion method (or) Long division method.

Type-1 : Partial Fraction Method

* Convert $X(z)$ to the positive powers of z.
* Bring $X(z)$ to $\frac{X(z)}{z}$.
* Apply partial fractional method
* Multiply with z.
* Take inverse z-transform based on Roc.

(a) Find the inverse z-transform of

$$X(z) = \frac{(z+4)z}{z(z^2-4z+3)}$$

Sol:

$$\frac{x(z)}{z} = \frac{(z+4)z}{z(z^2-4z+3)}$$

$$(z-1)(z-3)$$

$$\frac{X(z)}{z} = \frac{z+4}{z(z-1)(z-3)}$$

$$\frac{z+4}{z(z-1)(z-3)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-3}$$

$$z+4 = A(z-1)(z-3) + B(z)(z-3) + C(z)(z-1)$$

Put $z=1$

$5 = B(1)(-2)$

$5 = B(-2)$

$B = \frac{-5}{2}$

Put $z=3$

$7 = C(3)(2)$

$7 = C(6)$

$C = 7/6$

Put $z=0$

$H = A(-1)(-3)$

$H = A(3)$

$A = H/3$

$$\frac{X(z)}{z} = \frac{H/3}{z} - \frac{5/2}{z-1} + \frac{7/6}{z-3}$$

$\frac{H}{3} z^{-1} \left(\frac{z}{z}\right)$

$$x(z) = \frac{H}{3} z^{-1}\left(\frac{z}{z}\right)_1 - \frac{5}{2} z^{-1}\left(\frac{z}{z-1}\right)_{z \cdot a} + \frac{7}{6} z^{-1}\left(\frac{z}{z-3}\right.$$

$$x(n) = \frac{H}{3}\delta(n) - \frac{5}{2}\underbrace{(1)^n u(n)}_{a^n u(n)} + \frac{7}{6} 3^n u(n)$$

$$Z[\delta(n)] = 1$$

$$Z[u(n)] = \frac{z}{z-1}$$

$$Z[a^n u(n)] = \frac{z}{z-a}$$