ROC [Region of Convergence]

It is the region where z-transform converges.

* Gives an idea about values of z for which z-tranform can be calculated.
* To Determine causality of the system.

* To determine the stability of the system.

Q) Determine Z-transform of $x(n) = \{1, 2, 3, 4, 5, 0, 7\}$

$x(0) = 1 \quad x(4) = 5$

$x(1) = 2 \quad x(5) = 0$

$x(2) = 3 \quad x(6) = 7$

$x(3) = 4$

$$= \sum_{n=0}^{6} x(n) z^{-n}$$

$$= x(0) z^{0} + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3} + x(4) z^{-4} + x(5) z^{-5} + x(6) z^{-6}$$

$$= 1.1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 5z^{-4} + 7z^{-6}$$

$$x(n) = 1 + \frac{2}{z} + \frac{3}{z^2} + \frac{4}{z^3} + \frac{5}{z^4} + \frac{7}{z^6}$$

∴ ROC is the entire z-plane except at 0.

Q) Find the z-transform of $u(n)$

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Sol: $$\sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} 1 . z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + \dots + z^{-n}$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \dots$$

$$= (1 - 1/z)^{-1}$$

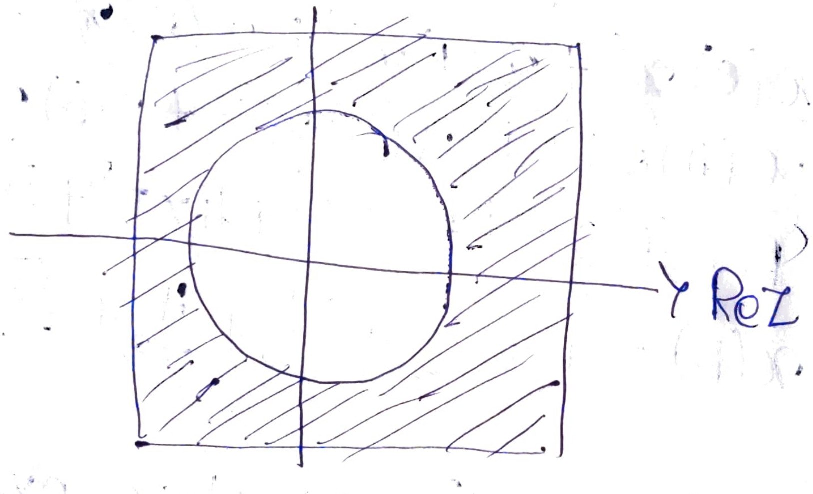
$$= \left(\frac{z-1}{z}\right)^{-1}$$

$$x(z) = \frac{z}{z-1}, \quad |z| > 1$$

$|z|$ - circle in z plane

$|z| = 1$ - circle of radius 1

$|z| > 1$ - Area outside the circle.



Q) $x(n)\ a^n u(-n-1)$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(-n-1) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n z^{-n}$$

$$= \sum_{n=1}^{\infty} a^{-n} z^{+n}$$

$n=1$

$= (a^{-1}z)^1 + (a^{-1}z)^2 + (a^{-1}z)^3 + \ldots$

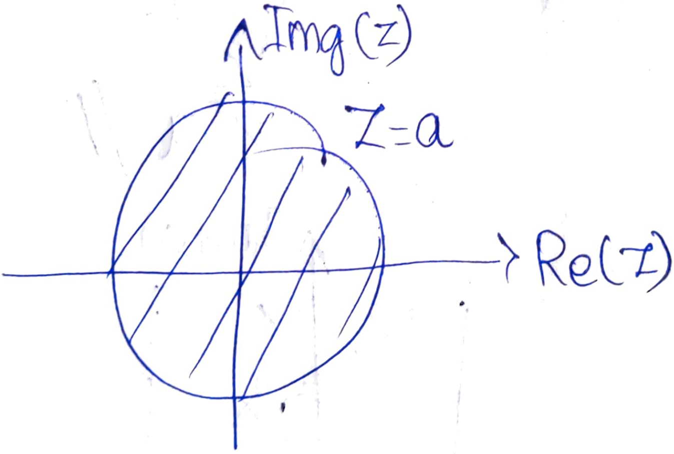
$= \cancel{z}a^{-1}[1 + (za^{-1}) + (za^{-2}) + \ldots]$

$= \dfrac{za^{-1}}{1 - z/a}$

$= \dfrac{z/a}{1 - z/a}$

$= \dfrac{z/a}{a - z/a}$

$x(z) = \dfrac{z}{a-z}, \quad |z| < a$



3) Find z-transform of $\delta(n)$.

$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$\delta(n) = \begin{cases} 1; & n=0 \\ 0; & n \neq 0 \end{cases}$

$z^0 = 1$

$= \sum_{n=0}^{\infty} x(n) z^{-n} = 1$ //

Q) Find the z transform of right hand sided sequence $x(n) = a^n u(n)$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n}$$

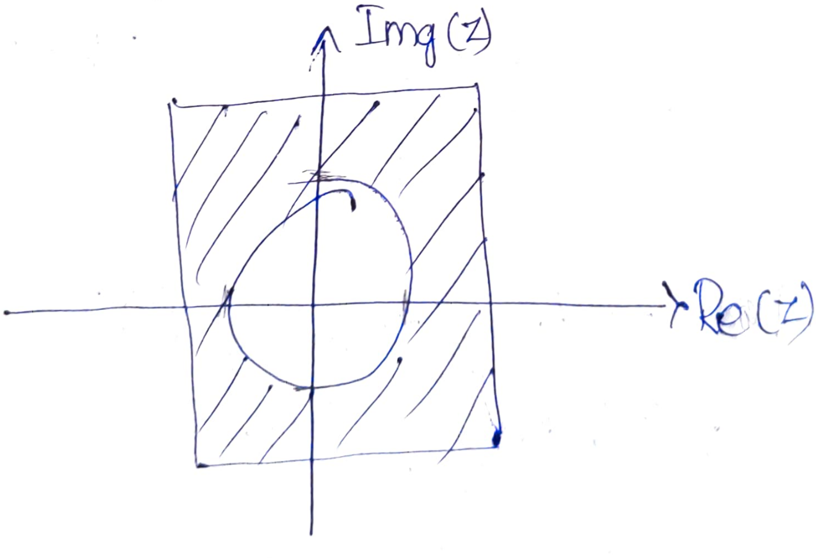
$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= 1 + (az^{-1})^1 + (az^{-1})^2 + \dots + (az^{-1})^n$$

$$= \frac{1}{1-az^{-1}} = \frac{1}{1-a/z}, \quad |z| > 0$$

$$= \frac{z}{z-a} \; //$$



a) Find the z-transform of both sided sequence.

$$x(n) = a^n u(n) + b^n u(-n-1)$$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) + b^n u(-n-1) z^{-n}$$

$$a^n u(n) = \frac{z}{z-a}$$

$$b^n u(-n-1) = \frac{z}{b-z}$$

$$\Rightarrow \frac{z}{z-a} + \frac{z}{b-z}$$

$$|z| > a, \quad |z| < b$$

$$b < |z| < a$$