UNIT-IV

Analysis of Discrete Time Signal

Z-Transform:

It is used for analysis of discrete time signals & systems.

Z-transform of $x(n)$ is defined as

$$Z[x(n)] = X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$$

Here Z is a complex variable

$$Z = r.e^{j\omega}$$

r is the magnitude of Z

ω is the phase angle of Z

$$\omega = \angle Z$$

Inverse Z-transform:

$$x(n) = \frac{1}{2\pi j} \oint_c X(v)\, v^{n-1} dv$$

Relationship between DTFT and Z-Transform

DTFT : $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

Z Transform: $X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$

$Z \rightarrow$ complex variable

$Z = r.e^{j\omega}$

$$Z[x(n)] = x(Z) = \sum_{n=-\infty}^{\infty} x(n) (re^{j\omega})^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, r^{-n} \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} [x(n)\, r^{-n}]\, e^{-j\omega n}$$

$$Z[x(n)] = DTFT\, [r^{-n} x(n)]$$

$$|Z| = r = 1$$

$$Z[x(n)] = DTFT\, [x(n)]$$

$$x(Z) = x(\omega) /_{Z = e^{j\omega}}$$

Types of Z-Transform:

(1) Bilateral or two sides Z-transform

$$x(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$$

(2) Unilateral or one sided Z-transform

$$x(Z) = \sum_{n=0}^{\infty} x(n) Z^{-n}$$