Elasticity

Unit-3

Twomarks:

Level 1:

1. Define elasticity.
2. Define hearing strain?
3. Define a Cantilever.
4. Give the applications of I-shape girders.
5. State Hooke’s law

Level 2:

1. Define torque.
2. Define a beam.
3. What is moment of force?
4. Explain neutral axis
5. Explain bending moment of beam.

Level 3:

1. What are the factors of hammering and annealing on elasticity of material?
2. Mention the factors affecting the elasticity of a material.
3. What do you infer from stress-strain diagram?
4. How do temperature and impurity in a material affect the elasticity of the materials?
5. What is meant by annealing?
6. Give the relation between the three modulii.

14 Marks:

Level 1:

1. Describe with necessary theory,the method to determine the Young's modulus of the material of a rectangular bar by uniform bending.

Level 2:

1. What is cantilever?Obtain an expression for the depression at the loaded end of a cantilever whose other end is fixed assuming that its own weight is not effective in bending.

2. Describe a experiment to determine the Young's modulus of a beam using bending of beams?

3. Derive an expression for the internal bending moment of a beam interms of radius of curvature.

Level 3:

1. i) Derive an expression for the elevation at the centre of a cantilever which is loaded at both ends.ii)Describe an experiment to determine Young's modulus of a beam by uniform bending.

UNIT-2b

ThermalPhysics

Onemark

Level-I

1. Thetransferofheatenergybetweenobjectsthatareinphysicalcontact occurs ________

a) Convection b)**conduction** c)Radiation d)Noneofthese

2. Thethermalconductivityinamaterialdecreaseswith____________

a) Area **b)Distancebetweenthe faces** c)Temperature d)Time

3. Metalshavethermalconductivitiesintherangeof

(a)<1(b)1-5(c)5-25**(d)20-400**

4. Thetemperaturegradientisgivenby______________

a) dx/dθ **b)dθ/dx** c)dθ/dt d)dt/dθ

5. Theheatflowingthroughtheobjectsinseries isgivenby______________

a) $Q=A(\theta_1-\theta_2)/\Sigma(x/K)$ b)$Q=A(\theta_1-\theta_2)/\Sigma(K/x)$ c)$Q=(\theta_1-\theta_2)/\Sigma(K/x)$ d)$Q= A/\Sigma(x/K)$

Level-II

1. Theco-efficientofthermalconductivityofaplatedependson

a) Area of the plate b)thicknessoftheplatec)temperaturedifference across the plate **d) Nature of material of plate**

2. withincreaseintemperature,thermalconductivityofametal ______________.

(a)Increases(b)Decreases(c)Either**(d)All,dependingon metal.**

3. Amongthefollowingwhichmaterialshavehighestthermalconductivity?

a) Silver b)copper c)glass d)Wood

4. Theheatlostperseconddueotoradiationisgivenby________________

a)Epδxθ b)Aδxρ c)dx/dθ d)dθ/dx

5. Thermalconductivityforgoodconductorisdeterminedby______________

a) Lee'sdiscmethodb)Radialheatflow method **c)Searle's method** d)Noneoftheabove

LEVEL-III

11) The heatflowingthroughthe bodies inparallelis givenby____________

a) $Q=(\theta_1-\theta_2)\Sigma K/x$ **b)** $\mathbf{Q=(\theta_1-\theta_2)\Sigma KA/x}$ c) $Q=(\theta_1-\theta_2)\Sigma K$

d) $Q=(\theta_1-\theta_2)\Sigma Kx$

12) The rateof heatloss of a bodyis ___________

a) directlyproportionaltothetemperatureb)directlyproportionaltotime

c)specificheatcapacityofthe body **d)All the above**

13) Thequantityofheatflowingradiallyperunittimethroughthewallofa spherical shell is ________

a) $\mathbf{Q=4\pi k(r_1r_2/r_2-r_1)(\theta_1-\theta_2)}$ b) $Q=k(r_1r_2/r_2-r_1)(\theta_1-\theta_2)$ c) $Q=(r_1r_2/r_2-r_1)(\theta_1-\theta_2)$

d) $Q=4\pi k(r_1r_2/r_2-r_1)(\theta_2-\theta_1)$

14) A 30cm long iron rod is heated, the temperature difference between the end is 65 degree Celsius, thermal conductivity of iron is $62Wm^{-1}K^{-1}$ and the areaofcrosssectionis$1cm^2$.Thentheamountofheatconductedthroughthe rod in 3 min is_____________

a) 467.48J b)440J c)290J **D)372J**

15) If 20J of heat is passing through a slab of area $90cm^2$ and the temperature differencebetweenthesidesis20Kandthecoefficientofthermalconductivity is $0.04Wm^{-1}K^{-1}$ the thickness of the slab is________________

a) $1.2x10^{-4}$m b) $2.4x10^{-4}$m **c)** $\mathbf{3.6x10^{-4}m}$ d) $4.8x10^{-4}$m

Twomarks

Level-I

1. Distinguishbetweenconductionandconvection.
2. Whatismeantbytemperaturegradient?
3. Definethermaldiffusivity.
4. Definecoefficientofthermalconductivity.
5. Give the methods of determining the thermal conductivity of good and bad conductors.
6. Whatissteadystate?

Level-II

1. Twobarsofcopperandseeloflength1mand0.5mrespectivelyandofco-efficientof thermalconductivity400W/mK and50W/mK arejoinedend to end. The free ends of copper and steel are maintained at 100°C and 0°C respectively. Calculate the temperature of Copper- steel Junction if Both bars have the Same area of Cross section.
2. The outer ends of two bars A&B are at 100°C and 50°C respectively. Calculatethetemperatureattheweldedjointiftheyhavethe samecross section and the same length and their thermal conductivities are in the ratioA:B=8.5.
3. Whatisthermalresistance?

LEVEL-III

1.If the Amount of heat conducted by ice is 3712.5J and the specific latent heatoficeis3.36x10^5JK$g^{-1}$$K^{-1}$.Findthemassoftheicethatmeltsperminute.

2. Calculate the amount of heat conducted by ice if the melted mass of the ice is5dueto180000Jofheatandthespecificlatentheatoficeis3.36x10^5JK$g^{-1}$$K^{-1}$.

14-
MarksLE
VEL-I

1. Deduceanexpressionfortheheatconductionalongauniformbar.Also obtain the steady state solution for it.
2. DescribeLee'sdiscmethodtofindtheco-efficientofthermalconductivity of a bad conductor.

LEVEL-II

1. Obtain an expression for the quantity of heat conducted radially out of a hollowcylinder.Usingthisexplainhowthethermalconductivityofrubber can be determined.
2. ObtainanExpressionforheatconductioninacompoundmediawhenthe bodies are in series and parallel.

Level-III

1. Deduceanexpressionforandmethodtodeterminethespecificheat capacity of a metal.
2. Explainamethodfordeterminingathermalconductivityofbadconductor.

1. Distinguishbetweenconductionandconvection.

   Conduction: It is the process in which the heat is transferred from hot end to cold end without actual movement of the particles.

   Convection: It is the process in which the heat is transmitted from hot end to cold end by the actual movement of particles.

2. Whatismeantbytemperaturegradient?

   The rate of fall of temperature with respect to the distance is called as temperature gradient. Ingeneral it is denotedas $-d\theta / dx$. The negative sign indicates the fall of temperature with the increase in distance.

3. Definethermaldiffusivity.

   It is defined as the ration of thermal conductivity to the thermal capacity per unit volume of the material. Since thethermal capacity is the product of specific heat capacity and density of the material, we can write

   $$h = k/\rho s \, m^2 s^{-1}$$

4. DefineNewton'slawofcooling.

The rate of loss of heat of a body is directly proportional to thetemperature difference between the body and its surrounding, of the same nature.

5. Definecoefficientofthermalconductivity.
   It is defined as the amount of heat conducted per second, normally across unit area of cross section maintained at unit temperature gradient.
   $K=Q.x/A(\theta1-\theta2)t Wm^{-1}K^{-1}$.

6. Givethemethodsofdeterminingthethermalconductivityofgoodandbad conductors.
   1. Searles'smethod
   2. Forbe'smethod
   3. Lee'sdiscmethod
   4. Radialflowmethod.

7. Definespecificheatcapacity.
   Itisdefinedastheamountofheatrequiredtoraisethetemperatureof unit mass of the substance through one Kelvin.

$$S=Q/m\theta JKg^{-1}K^{-1}$$

8. Whyshouldthespecimenusedtodeterminethermalconductivityofabad conductor should have a larger area and smaller thickness?
   Forabadconductorwithsmallerthicknessandlargerareaofcrosssection the amount of heat conducted will be more.

9. Whatismeantbyradialflowofheatmethod?Giveits uses.
   In radial flow of heat method heat flows from inner sphere or cylinderalong the radius and hence the heat is radiated radially across all layersthusitiscalledradialflowmethod.Thismethodisusefulindeterminingthe thermal conductivity of bad conductors taken in the powder form.

10. Whatissteadystate?

When a solid bar is heated at one end, each particle absorbs some heat, raise its own temperature loses a little by radiation etc and passes on the resttothenext,astageisreachedwheneachparticlehastakenitsfulland cannot absorb any more heat.

11. WhatarethelimitationsofNewton'slawof cooling?

*Thetemperaturedifferencebetweenthehotbodyandsurrounding should be low.

*Theheatlossisonlybyradiationandconvection.

*Thetemperatureofhotbodyshouldbeuniformthoughout.

12. WhataretheusesofNewton'slawof cooling?

ThespecificheatcapacityoftheliquidisdeterminedbyusingNewton'slaw of cooling.

13. Howheatconductionandelectricalconductionanalogoustoeachother?

S.No	Heat conduction	ElectricalConduction
1	Heatisconductedfromapoint of higher temperature to a pointoflowertemperature.	Electricity is conducted from a pointahigherpotentialtoapoint atlower potential.
2	Inmetalsheatconductionis mainlyduetofreeelectron.	In metals electrical conduction is duetofreechargecarriersnamely electrons.
3	Theabilitytoconductheatis measured by thermal conductivity.	Theabilitytoconductelectricityis measured by electrical conductivity.

14. Whatisthermalresistance?

Thethermalresistanceofabodyisameasureofitsoppositiontotheflow of heat through it.

14MARK QUESTIONS

1. ➢ **Derive a differential equation (second order) to describe the heat conduction along a uniform bar. Hence obtain the steady state solution of it.** **(Dec.1997)**

➢ **Derive the equation for heat conduction along a bar and solve it for steady state condition.** **(Dec.1998)**

➢ **Derive the equation for one-dimensional flow of heat and solve it under steady state condition.** **(Nov.2001)**

ANSWER

Rectilinear flow of heat along an uniform bar One dimensional flow of heat

Let us consider the bar AB of uniform area of cross section 'A' exposed to air, lying along the x axis (one dimensional). Let one end of the bar be heated with the help of steam chamber as shown in the fig.C-1

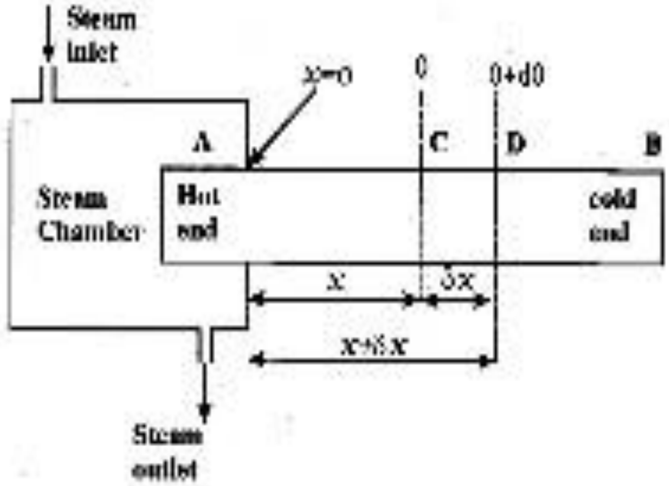

Fig. C-1

Let us consider the plane C and D at a distance x and $x+\delta x$ from the hot end respectively. Let θ and $\theta+d\theta$ be the temperature at C and D respectively.

Then

The temperature gradient at Plane 'C' $= \dfrac{d\theta}{dx}$

Here the excess of temperature at Plane D $= \left(\theta + \dfrac{d\theta}{dx}\delta x\right)$

$\therefore$ Temperature gradient at Plane D $= \dfrac{d}{dx}\left(\theta + \dfrac{d\theta}{dx}\delta x\right)$

The amount of heat conducted per second at

$$C = Q_1 = -KA\frac{d\theta}{dx} \qquad \text{........(1)}$$

The amount of heat conducted per second at

$$D = Q_2 = -KA\frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right) \qquad \text{.......(2)}$$

The amount of heat gained by the rod per second between C and D is

$$Q = Q_1 - Q_2$$

Substituting from equations (1) and (2) we get

$$Q = -KA\frac{d\theta}{dx} - \left[-KA\frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right)\right]$$

$$Q = KA\frac{d^2\theta}{dx^2}\cdot\delta x \qquad \ldots\ldots.(3)$$

Before steady state is reached

Before the steady state is reached the amount of heat 'Q' is used in two ways.

1) Part of the heat is used to raise the temperature of the rod.

(i.e) If $d\theta/dt$ is the rise in temperature per second, ρ is the density of the rod and 'S' is the specific heat, Then

Amount of heat used per second to raise the temperature of rod.

$$Q_3 = \text{mass} \times \text{specific heat} \times \frac{d\theta}{dt}$$

$$= A\,\delta x\,\rho\,s\,\frac{d\theta}{dt} \qquad \ldots\ldots..(4)$$

Where mass = Volume × density

$$= A\,\delta x\,\rho$$

2) Rest of the heat is radiated from the surface of the rod.

(i.e) If E is the emissive power and P is the perimeter then we can write

The amount of heat lost per second due to radiation = $EP\,\delta x\,\theta$ ....(5)

∴ Before steady state is reached, the amount of heat used to raise the temperature.

$$Q = Q_3 + Q_4$$

Substituting from equation (4) and equation (5), we get

$$Q = A\,\delta x\,\rho s\frac{d\theta}{dt} + EP\,\delta x\,\theta \qquad \ldots\ldots..(6)$$

Substituting equation (3) in (6) we have

$$KA\frac{d^2\theta}{dx^2}\delta x = A\,\delta x\,\rho s\frac{d\theta}{dt} + EP\,\delta x\,\theta$$

$$\frac{d^2\theta}{dx^2} = \frac{\rho s}{K}\frac{d\theta}{dt} + \frac{EP}{KA}\theta \qquad \text{........(7)}$$

This is the general equation for the flow of heat in one dimension. Here $K/\rho s$ is called as thermal diffusivity(h) of the bar.

After steady state is reached and if the bar is of infinite length

After the steady state is reached, the rod does not require further heat to raise to temperature (i.e) the temperature will become constant at this stage.

$$\text{i.e.} \quad \frac{d\theta}{dt} = 0$$

$\therefore$ Equation (7) becomes $\frac{d^2\theta}{dx^2} = \frac{EP}{KA}\theta$

Assuming $\frac{EP}{KA} = \mu^2$

We can write $\frac{d^2\theta}{dx^2} = \mu^2\theta$ ....(8)

The general solution for equation (8) is

$$\theta = Ae^{\mu x} + Be^{-\mu x} \qquad \text{........(9)}$$

Where A and B are the arbitary constants, which can be determined by applying boundary conditions.

If the Bar is assumed to be infinite length then the boundary conditions are

i) At $x = 0$; $\theta = \theta_0$

Equation (9) becomes $\theta_0 = A+B$

ii) At $x = \infty$; $\theta = 0$ (Since it is assumed that the bar is of infinite length, the excess of temperature at the other end is zero).

Equation (9) becomes $0 = Ae^{\infty}$

Here $e^{\infty} \neq 0$ $\therefore$ 'A' should be equal to zero (i.e.) $A=0$

Substituting A=0 in equation (10) we have

$$\theta_0 = 0 + B \quad \text{(or)} \quad B = \theta_0$$

Substituting the values of A and B in equation(9)

$$\theta = \theta_0 e^{-\mu x} \qquad \text{........(11)}$$

Equation (11) represents the excess of temperature of any point / plane at a distance x from the hot end, after steady state is reached, which is an exponential function.

Therefore a graph is plotted between x and θ, which gives raise to exponential form as shown in the fig.C-2.

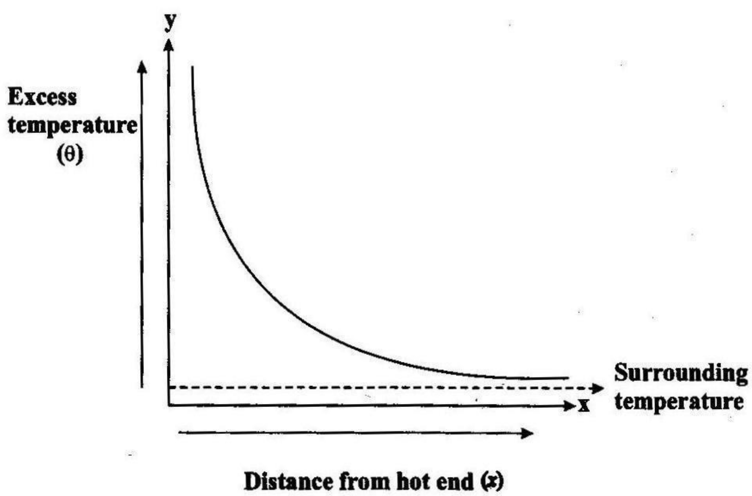

Fig. C-2

\fter steady state is reached and if the bar is of finite length /) (covered with insulating materials)

When the bar of finite length (l) is covered by insulating materials as hown in fig.C-3, then the heat lost due to radiation will be very small

∴ Emissive power (E) = 0; and

After steady state, $\frac{d\theta}{dt} = 0$

Therefore equation(7) becomes $\frac{d^2\theta}{dx^2} = 0$

Intergrating twice, we can write $\theta = Cx + D$ ....(12)

where C and D are the arbitrary constants which can be evaluated by applying the boundary conditions.

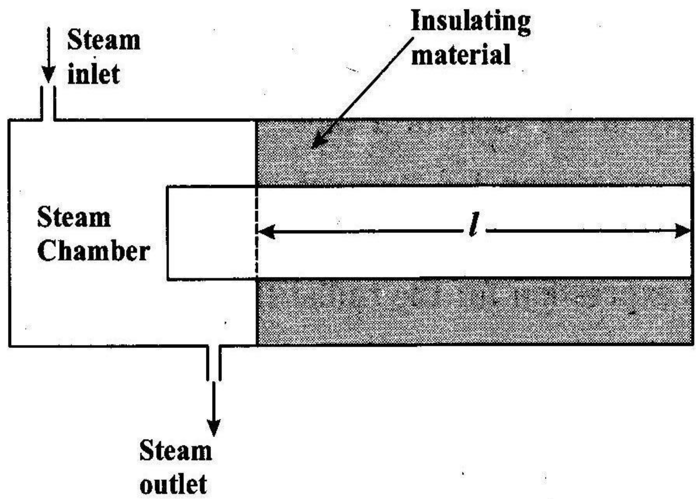

Fig. C-3

(i) At $x = 0$; $\theta = \theta_0$

Equations (12) becomes $\theta_0 = D$

(ii) At $x = l : \theta = \theta_1$ (where θ_1 is the excess of temperature at the end of bar.)

∴ Equation 12 becomes

$$\theta_1 = \ell C + \theta_0 \quad (\text{Since } D = \theta_o)$$

(or)

$$C = \frac{\theta_1 - \theta_o}{\ell}$$

Substituting the values of C and D in equation (12), we get

$$\theta = \left(\frac{\theta_1 - \theta_o}{\ell}\right) x + \theta_0 \qquad .....(13)$$

Equation (13) represents the excess of temperature of any point / plane at a distance x from the hot end, when it is covered by insulator (or) insulating material.

2.

- **Obtain an expression for the quantity of heat conducted radially out of a hollow cylinder. Using this, explain how the thermal conductivity of rubber can be determined. (Dec.1997)**
- **Discuss with necessary theory the method of determining the thermal conductivity in the form of a tube. (Nov.1998)**
- **Derive an expression for thermal conductivity of the material of a thick pipe through which a hot liquid is flowing. (Nov.2001)**
- **Derive an expression for the radial flow of heat through a cylindrical tube. (May,2003)**

ANSWER

Radial Flow of Heat

In this method heat flows from inner sphere (or) cylinder along the radius and hence the heat is radiated radially across all layers, thus called as radial flow method. This method is useful in determining the thermal conductivity of bad conductors taken in the powder form.

Description: This method is useful in finding the thermal conductivity of refrigerator pipings, steam pipe, etc. Let us consider a thick cylindrical tube of length 'l' inner radius r_1 and the outer radius r_2 as shown in fig. C-4. The steam can be passed through the centre of the shell.

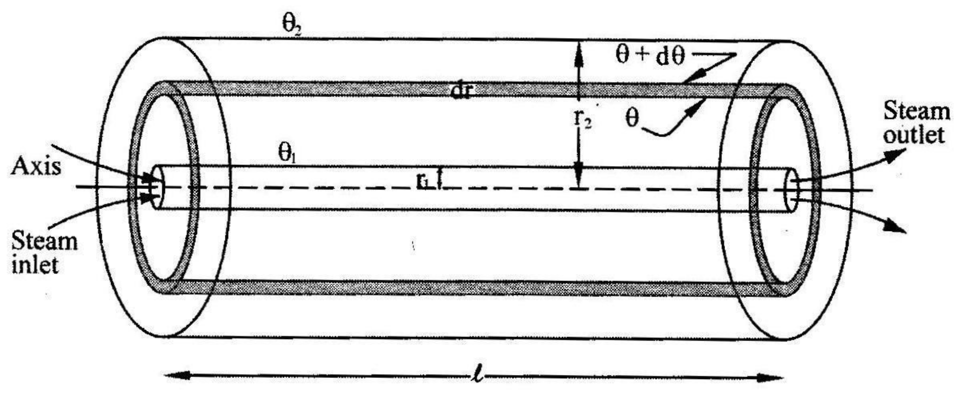

Fig. C-4

Working: Steam is allowed to pass through the axis of the cylindrical shell. The heat flows from the inner surface to the outer surface radially. After the steady state is reached, the temperature at the inner surface is noted as θ_1 and on the outer surface is noted as θ_2.

Calculation: The cylinder may be considered to consists of a large number of co-axial cylinders of increasing radii. Consider such an elemental cylindrical shell of the thickness *dr* at a distance '*r*' from the axis. Let the temperatures of inner and outer surfaces of the elemental shell be θ and $\theta + d\theta$. Then,

The amount of heat conducted *per second* $Q = -KA\frac{d\theta}{dr}$.

Here Area of cross section $A = 2\pi r l$

$$\therefore \qquad Q = -2\pi r l K \frac{d\theta}{dr}$$

Rearranging we have

$$\frac{dr}{r} = \frac{-2\pi \, lK}{Q} . d\theta \qquad \text{.......(1)}$$

$\therefore$ The thermal conductivity of the whole cylinder can be got by, integrating eqn.(1) within the limits r_1 to r_2 and θ_1 to θ_2,

$$\text{(i.e.)} \qquad \int_{r_1}^{r_2} \frac{dr}{r} = -\frac{2\pi l K}{Q} \int_{\theta_1}^{\theta_2} d\theta$$

$$[\log_e]_{r_1}^{r_2} = \frac{-2\pi l K}{Q}(\theta_2 - \theta_1)$$

$$\text{(or)} \qquad \log_e \frac{r_2}{r_1} = \frac{2\pi \, l K}{Q}(\theta_1 - \theta_2)$$

Rearranging we get,
$$K = \frac{Q . \log_e (r_2 / r_1)}{2\pi l (\theta_1 - \theta_2)}$$

$$\text{(or)} \qquad K = \frac{Q \times 2.3026 \times \log_{10}(r_2/r_1)}{2\pi l(\theta_1 - \theta_2)} \qquad W\,m^{-1}\,K^{-1}$$

By knowing the values in RHS, the thermal conductivity of the given material can be found.

Thermal Conductivity of a Rubber tube

Description: It consists of a calorimeter, stirrer with a thermometer. The setup is kept inside the woodenbox. The space between the calorimeter and the box is filled with insulating materials such as cotton, wool, etc. to avoid radiation loss, as shown in fig. C-5.

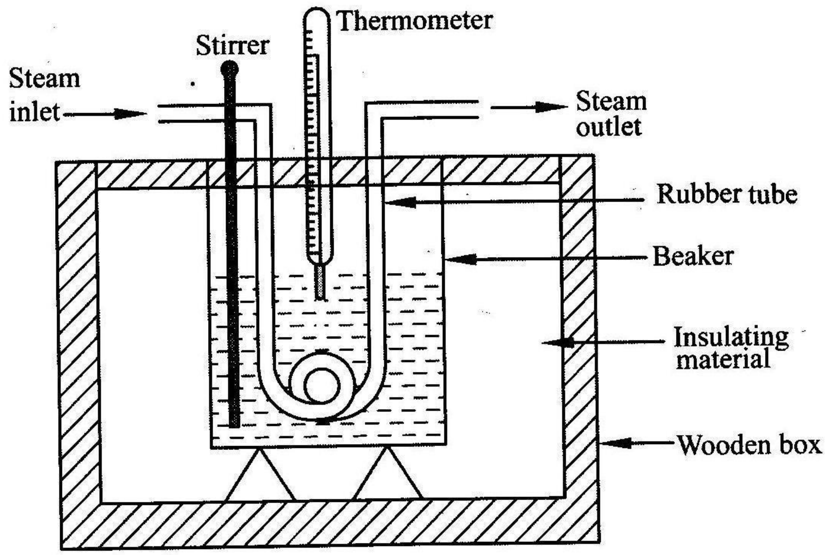

Fig. C-5

Working: The empty calorimeter is weighted, let it be (w_1). It is filled with two third of water and is again weighed, let it be w_2. A known length of rubber tube is immersed inside the water contained in the calorimeter. Steam is passed through one end of the rubber tube and let out through the other end of the tube. The heat flows from the inner layer of the rubber tube to the outer layer and is radiated. The radiated heat is gained by the water in the calorimeter. The time taken for the steam flow to raise the temperature of the water about 10°C is noted, let it be 't' seconds.

Observation and Calculation:

Let w_1 → Weight of calorimeter

w_2 → Weight of calorimeter and water.

$w_2 - w_1$ → Weight of the water alone

θ_1 → Initial temperature of the water

θ_2 → final temperature of the water

$\theta_2 - \theta_1$ → Rise in temperature of the water

θ_S → Temperature of the steam

l → Length of the rubber tube (immersed)

r_1 → Inner radius of the rubber tube

r_2 → Outer radius of the rubber tube

s_1 → Specific heat capacity of the calorimeter

s_2 → Specific heat capacity of the water

θ_3 → Average temperature of the rubber tube.

$$\text{(i.e)} \quad \theta_3 = \frac{\theta_1 + \theta_2}{2}$$

Then from the theory of cylindrical shell method the amount of hea conducted by the rubber tube *per second* is given by

$$Q = \frac{K 2\pi l (\theta_S - \theta_3)}{\log_e (r_2/r_1)} \qquad \text{..........(1)}$$

The amount of heat gained by calorimeter *per second* $= \dfrac{w_1 s_1 (\theta_2 - \theta_1)}{t}$

The amount of heat gained by water *per second* $= \dfrac{(w_2 - w_1) s_2 (\theta_2 - \theta_1)}{t}$

∴ The amount of heat gained by the water and calorimeter *per second.*

$$Q = \frac{(w_2 - w_1) s_2 (\theta_2 - \theta_1) + w_1 s_1 (\theta_2 - \theta_1)}{t}$$

(or)
$$Q = \frac{(\theta_2 - \theta_1)[w_1 s_1 + (w_2 - w_1) s_2]}{t} \qquad \text{.....(2)}$$

Under steady state

The amount of heat conducted by the rubber per second	=	The amount of heat gained by the water and the calorimeter per second

Therefore we can write eqn.(1) = eqn.(2)

$$\text{(i.e)} \quad \frac{K 2\pi l (\theta_S - \theta_3)}{\log_e (r_2/r_1)} = \frac{(\theta_2 - \theta_1)[w_1 s_1 + (w_2 - w_1) s_2]}{t}$$

Since $\theta_3 = \dfrac{(\theta_1 + \theta_2)}{2}$ we can write,

$$K = \frac{(\theta_2 - \theta_1) \log_e (r_2/r_1) [w_1 s_1 + (w_2 - w_1) s_2]}{2\pi l t \left[\theta_S - \frac{(\theta_1 + \theta_2)}{2}\right]} \quad Wm^{-1}K^{-1}$$

By substituting the values in RHS, the thermal conductivity of the rubber can be determined.

3. ➤ **Describe Lees' disc method to find the co-efficient of thermal conductivity of a bad conductor.** **(May,2003)**

ANSWER

Description:

It is a simple method used to determine the thermal conductivity of bad conductors like, rubber, glass, ebonite etc. The material whose thermal conductivity is to be determined should be taken in the form of two thin discs D_1 and D_2 about 10 cm in diameter and 2 to 3 mm of thickness. The disc 'D_1' is sandwitched between two copper plates C_1 and C_2. The disc 'D_2' is sandwitched between two copper plate C_3 and C_4 as shown in fig.C-6 In order to have good thermal contact, all the surfaces, are coated with glycerine.

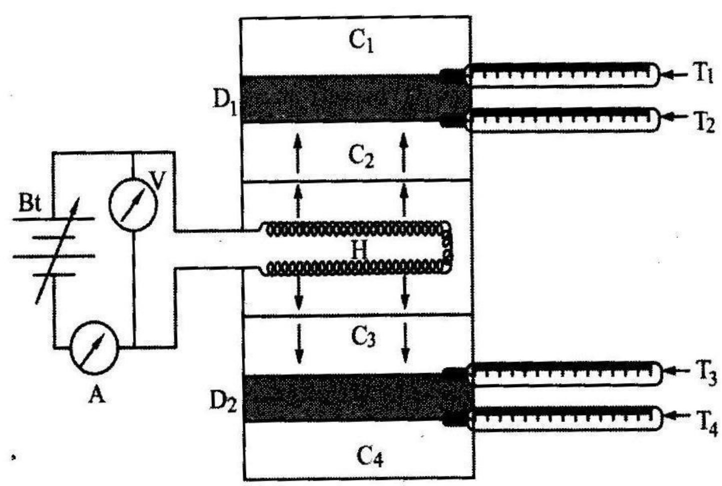

Fig. C-6

The copper discs can be heated using a heating coil (H) placed at the centre. The temperatures at the interface can be noted by the thermometers T_1, T_2, T_3 and T_4.

Working:

A steady current (*I*) is passed through the heating coil (*H*). The coil is heated and the heat spreads to the copper disc's. After a steady state is reached, the thermometer indicate constant (but different) readings. Let the temperatures indicated by the thermometers T_1, T_2, T_3 and T_4 be $\theta_1, \theta_2, \theta_3$ & θ_4 respectively.

Calculation:

Let the thickness of disc D_1 be x_1 and D_2 be x_2 then

Heat passing through D_1 *per second* $= \dfrac{KA(\theta_2 - \theta_1)}{x_1}$ (Since $\theta_2 > \theta_1$)

Heat passing through D_2 *per second* $= \dfrac{KA(\theta_3 \text{ to } \theta_4)}{x_2}$ (Since $\theta_3 > \theta_4$)

$\therefore$ Heat passing through D_1 and D_2 *per second* $= KA\left[\dfrac{(\theta_2 - \theta_1)}{x_1} + \dfrac{(\theta_3 - \theta_4)}{x_2}\right]$ .....(1)

Heat produced by the heater coil *per second* = EI ....(2)

Where E is the potential difference across the coil.

Under steady state

The heat produced by the heater coil per second	=	*The heat gained by the disc D_1 and D_2 per second*

$\therefore$ We can write eqn.(1) = eqn.(2)

(i.e.) $$KA\left[\frac{(\theta_2 - \theta_1)}{x_1} + \frac{(\theta_3 - \theta_4)}{x_2}\right] = EI$$

$\therefore$ $$K = \frac{EI}{A}\left[\frac{(\theta_2 - \theta_1)}{x_1} + \frac{(\theta_3 - \theta_4)}{x_2}\right]^{-1} \quad Wm^{-1}K^{-1}$$

Thus the thermal conductivity of the bad conductor can be determined from the above formula.

7. ➤ **Derive an expression for the quantity of heat flow through a metal slab whose faces are kept at two different temperatures. Use this expression to determine the thermal conductivity of a bad conductor by Lees' disc method.** **(Nov.2002)**

ANSWER

Description:

The given bad conductor (*B*) is shaped with the diameter as that of the circular slab (or) disc '*D*'. The bad conductor is placed inbetween the steam chamber (*S*) and the disc (or) slab (*D*), provided the bad conductor, steam chamber and the slab should be of same diameter. Holes are provided in the steam chamber (*S*) and the disc (or) slab (*D*) in which thermometer are inserted to measure the temperatures. The total arrangement is hanged over the stand as shown in fig. C-7.

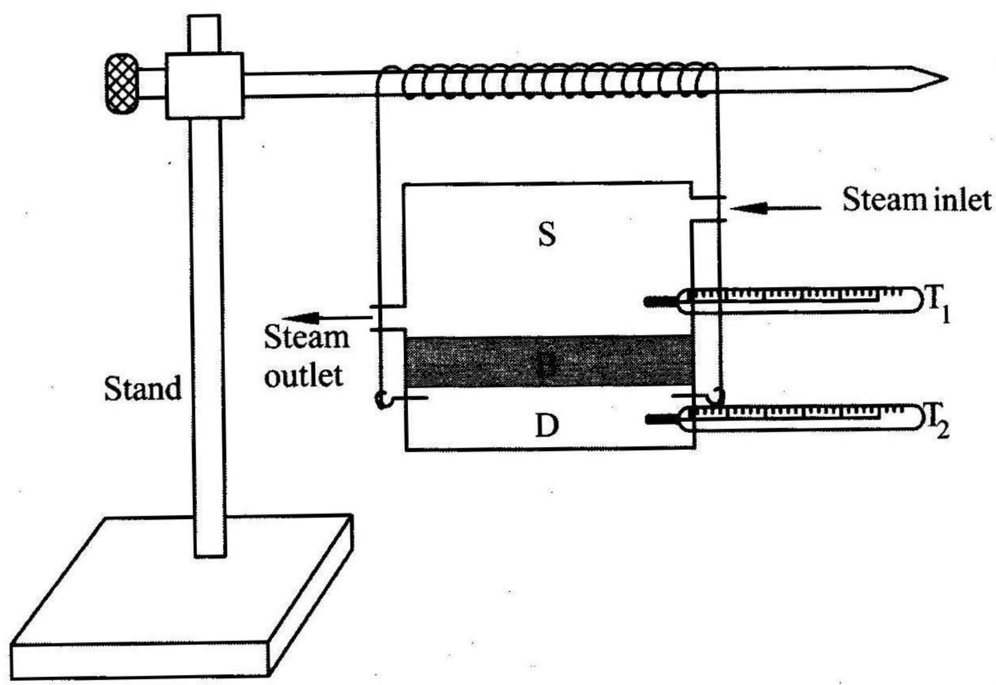

Fig. C-7

Working:

Steam is passed through the steam chamber till the steady state is reached (i.e) the thermometer show constant temperature. Let the temperature of the steam chamber (hot end) and the disc or slab (cold end) be θ_1 and θ_2 respectively.

Calculation:

Let 'x' be the thickness of the bad conductor(B), 'm' is the mass of the slab, 's' be the specific heat capacity of the slab. 'r' is the radius of the slab and 'h' be the height of the slab, then

Amount of heat conducted by the bad conductor *per second* $= \dfrac{KA(\theta_1 - \theta_2)}{x}$

Since the Area of cross section is $= \pi r^2$.

Amount of heat conducted *per second* $= \dfrac{K\pi r^2(\theta_1 - \theta_2)}{x}$ .....(1)

The amount of heat lost by the slab *per second* $= m \times s \times$ Rate of cooling

$= ms R_C$ ....(2)

Under steady state

The amount of heat conducted by the bad conductor(B) per second	=	Amount of heat lost by the slab (D) per second

Therefore, we can write eqn.(1) = eqn.(2)

$$\frac{K\pi r^2(\theta_1 - \theta_2)}{x} = msR_C$$

$$K = \frac{m\, s\, x\, R_C}{\pi r^2(\theta_1 - \theta_2)}$$ ....(3)

To find the rate of cooling (R_C)

In Eqn.(3) R_C represents the rate of cooling of the slab along with the steam chamber. To find the rate of cooling for the slab alone, the bad conductor is removed and the steam chamber is directly placed over the slab and heated.

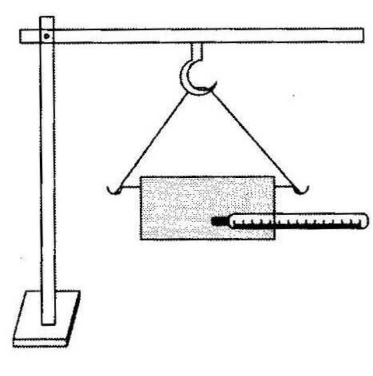

Fig. C-8

When the temperature of the slab attains 5°C higher than θ_2, the steam chamber is removed. The slab is allowed to cool, as shown in fig. C-8 simultaneously starting a stop watch.

A graph is plotted taking time along '*x*' axis and temperature along '*y*' axis, the rate of cooling for the slab alone (i.e) $\left(\frac{d\theta}{dt}\right)$ is found from graph, as shown fig.C-9.

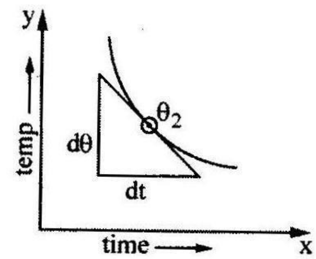

Fig. C-9

The rate of cooling is directly proportioal to the surface area exposed.

Case (i)

Steam chamber and bad conductor are placed over slab, in which radiation takes place from the bottom surface of area (πr^2) of the slab and the sides of the slab of area ($2\pi r h$).

$$\therefore\ R_C = 2\pi r^2 + 2\pi r h$$

(or)

$$R_C = \pi r(r + 2h) \qquad \text{.........(4)}$$

Case (ii)

The heat is radiated by the slab alone, (i.e) from the bottom of area (πr^2), top surface of the slab of area πr^2 and also through the sides of the slab of area $2\pi r h$.

$$\therefore\ \left(\frac{d\theta}{dt}\right)_{\theta_2} = \pi r^2 + \pi r^2 + 2\pi r h$$

(or)

$$\left(\frac{d\theta}{dt}\right)_{\theta_2} = 2\pi r^2 + 2\pi r h$$

$$\left(\frac{d\theta}{dt}\right)_{\theta_2} = 2\pi r(r + h) \qquad \text{.........(5)}$$

From eqn.(4) and eqn.(5)

$$\frac{R_C}{(d\theta/dt)_{\theta_2}} = \frac{\pi r\,(r + 2h)}{2\pi r(r + h)}$$

(or) $$R_C = \frac{(r+2h)}{2(r+h)}\left(\frac{d\theta}{dt}\right)_{\theta_2} \qquad \text{..........(6)}$$

Substituting eqn.(6) in eqn.(3) we have

$$K = \frac{ms\,x\left(\frac{d\theta}{dt}\right)_{\theta_2}.(r+2h)}{\pi r^2(\theta_1-\theta_2)\,2(r+h)} \ \text{Wm}^{-1}\text{K}^{-1}$$

Hence, thermal conductivity of the given bad conductor can be determined from the above formula.