Solving the Difference Equations using z-Transforms

(2) Solve the difference equation $y(n+3) - 3y(n+1) + 2y(n) = 0$ is gn that $y(0) = 4, y(1) = 0, y(2) = 8$

Taking z BS

$$z[y(n+3)] - 3z[y(n+1)] + 2z[y(n)] = 0$$

$$z^3 F(z) - z^3 y(0) - z^2 y(1) - z y(2) - 3[z f(z) - z y(0)] + 2F(z) = 0$$

$$\Rightarrow z^3 F(z) - z^3(4) - z^2(0) - z(8) - 3z F(z) + 3z(4) + 2 F(z) = 0$$

$$\Rightarrow z^3 f(z) - 3z f(z) + 2 F(z) = 4z^3 + 8z - 12z$$

$$F(z)[z^3 - 3z + 2] = 4z^3 - 4z$$

$$F(z) = 4z^3 - 4z \Big/ z^3 - 3z + 2$$

$$F(z) = \frac{4z^3 - 4z}{z^3 - 3z + 2} = \frac{4z^3 - 4z}{(z+2)(z-1)^2}$$

$$\frac{F(z)}{z} = \frac{4z^2 - 4}{(z+2)(z-1)^2} = \frac{A}{z+2} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

put $z = -2 \Rightarrow A = 4/3$

put $z = 1 \Rightarrow C = 0$

put $z = 0 \Rightarrow 8/3$

$$\frac{F(z)}{z} = \frac{4z^2 - 4}{(z+2)(z-1)^2} = \frac{4/3}{z+2} + \frac{8/3}{z-1} \Rightarrow F(z) = \frac{4}{3}\left(\frac{z}{z+2}\right) + \frac{8}{3}\left(\frac{z}{z-1}\right)$$

$$[F(z)] = \frac{4}{3}z\left(\frac{z}{z+2}\right) + \frac{8}{3}z\left(\frac{z}{z-1}\right).$$

$$= \frac{4}{3}(-2)^n + \frac{8}{3}(1)^n.$$

③ Solve the difference equ. $y(n) + 3y(n-1) - 4y(n-2) = 0$, $n \geq 2$ is gn. that $y(0) = 3$, $y(1) = -2$.

$$y(n) + 3y(n-1) - 4y(n-2) = 0$$

Replace n by $n+2$.

$$y(n+2) + 3y(n+1) - 4y(n) = 0$$