TYPE - II [Heat flows in y direction, i.e., $0<y<l$.]

The boundary conditions are:

(i) $u(x,0)=0$

(ii) $u(x,l)=0$

(iii) $u(0,y)=0$

(iv) $u(l,y)=f(x), 0<y<l$.

The suitable soln. is $u(x,y)=(Ae^{px}+Be^{-px})(C\cos py+D\sin py)$

) A square plate bdd. by the lines $x=0$, $y=0$, $x=b$ & $y=b$ its faces are insulated. The temp. along the lower vertical edges is gn. by $u(b,y)=y(b-y)$, $0<y<b$, while the other three edges kept at $0°$. Find the steady state temp. in the plate.

Soln: The boundary cdtns. are

(i) $u(x,0)=0$

(ii) $u(x,b)=0$

(iii) $u(0,y)=0$

(iv) $u(b,y)=y(b-y)$.

The suitable soln. is

$$u(x,y)=(Ae^{px}+Be^{-px})(C\cos py+D\sin py)$$

Apply (i) $u(x,0)=(Ae^{px}+Be^{-px})C$

$$\boxed{0=C.}$$

$\therefore u(x,y) = (Ae^{px} + Be^{-px}) D \sin py$. (

Apply (ii), $u(x,b) = (Ae^{px} + Be^{-px}) D \sin pb$.

$0 = \sin pb\, D\, (Ae^{px} + Be^{-px})$

$\Rightarrow D \neq 0, \ \sin pb = 0$

$p = \frac{n\pi}{b}$

$\therefore u(x,y) = \left(Ae^{\frac{n\pi}{b}x} + Be^{-\frac{n\pi}{b}x}\right) D \sin \frac{n\pi}{b} y.$

Apply (iii), $u(0,y) = (A+B) D \sin \frac{n\pi}{b} y$.

$0 = A+B \Rightarrow \boxed{B = -A}$

$\therefore u(x,y) = AD\left(e^{\frac{n\pi}{b}x} - e^{-\frac{n\pi}{b}x}\right) \sin \frac{n\pi}{b} y.$

general soln. $u(x,y) = \sum_{n=1}^{\infty} A_n \sinh \frac{n\pi}{b} x \sin \frac{n\pi}{b} y$.

$y(b-y) = \sum_{n=1}^{\infty} A_n \sinh n\pi \sin \frac{n\pi}{b} y$:

$y(b-y) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi}{b} y, \quad B_n = A_n \sinh n\pi$

$B_n = \frac{2}{b} \int_0^b y(b-y) \sin \frac{n\pi}{b} y \, dy$.

$= \frac{4l^2}{n^2\pi^2} [1-(-1)^n]$.

$$A_n = \frac{4l^2}{n^2\pi^2} \frac{[1-(-1)^n]}{\sinh n\pi}$$

$$\therefore u(x,y) = \sum_{n=1}^{\infty} \frac{4l^2}{n^2\pi^2} \frac{[1-(-1)^n]}{\sinh n\pi} \sinh \frac{n\pi}{b} x \sin \frac{n\pi}{b} y$$