Derivation of Planck's Radiation law

Planck considered the black body radiations (in the hohlraum) to consist of linear oscillators of molecular dimensions and that the energy of a linear oscillator can assume only the discrete values

0,hv,2hv,3hv....nhv

If $N_{0,}$ N_1,N_2,...are the number of oscillators per unit volume of the hologram possessing energies0,hv,2hv....respectively, then the total number of oscillators N per unit volume will be

$$N = N_0+N_1+N_2+.....$$

But the number of oscillators, N_r having energy E_r is given by (Maxwell's formula)

$$N_r = N_0\, e^{-E/kT}$$

Putting these values in eqn., we get

$$N = N0 + N0\, e^{-hv/kT} + N0\, e^{-2hv/kT} + ...N0\, e^{-nhv/kT}$$

$$= N0(1+ e^{-hv/kT} + e^{-2hv/kT} +.....)$$

$$= \frac{N0}{1-e^{-hv/kT}} \quad [\because (1-e^{-x})^{-1} = 1+e^{-x}+e^{-2x}+....]$$

The total energy of N oscillators will be

$$\begin{aligned}
E &= N_1 \cdot hv + N_2 \cdot 2hv + N_3 \cdot 3hv + ...\\
&= hv(N_1+2N_2+3N_3+....)\\
&= hv(N_0 e^{hv/kT} + 2N_0 e^{2hv/kT} + 3N_0 e^{-3hv/kT} + ...)\\
&= N_0 hv(e^{-hv/kT} + 2e^{-2hv/kT} + 3e^{-3hv/kT} + ...)\\
&= N_0 hve^{-hv/kT}(1+2e^{-hv/kT} + 3e^{-2hv/kT} + ...)\\
&= N_0 hve^{-hv/kT}(1+2x_3x^2+....) \quad (where x = e^{-hv/kT})\\
&= N_0 hve^{-hv/kT}(1-x)^{-2}\\
&= \frac{N_0 hve^{-hv/kT}}{(1-x^2)} = N_0 hv \frac{e^{-hv/kT}}{(1-e^{-hv/kT})^2}
\end{aligned}$$

Hence the average energy per oscillator is

$$\begin{aligned}
E &= \frac{E}{N} = N_0 hv \frac{e^{-hv/kT}}{(1-e^{-hv/kT})^2} \times \frac{1-e^{-hv/kT}}{No}\\
&= \frac{hve^{-hv/kT}}{1-e^{-hv/kT}} = \frac{hv}{e^{-hv/kT}-1}
\end{aligned}$$

(on dividing numerator and denominator bye-hv/Kt)

Thus we see that the average energy of the oscillator is not Kt (as given by classical theory)but equal to hv/(ehv/kt-1) according to Planck's quantum theory.

Further, it can be deduced that he number of oscillators per unit volume having frequent in the range of v and v+dv is equal to

$$\frac{8\pi v^2 dv}{c^3}$$

Hence the average energy per unit value (i.e., energy density) inside the enclosure is obtained by multiplying with i.e. it is given by

$$E_V d_v = \frac{8\pi v^2 dv}{c^3} \times \frac{hv}{e^{hv/kT} - 1}$$

Putting $V = \frac{c}{\lambda}$ and $= -\frac{c}{\lambda^2} d\lambda$,the average energy per unit volume in the enclosure of the wave lights between λ and λ +d λ will be

$$E\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/kT} - 1} d\lambda$$

Or the energy radiated by the black body corresponding to wavelength λ is

$$E_\lambda \frac{8\pi hc}{\lambda^5} \frac{1}{e^{-hc/\lambda kT} - 1}$$

Which is Planck's radiation law or Pluck's distribution law?

The above equation is also quite often written in the form

$$E_\lambda = c1^{\lambda - 5} \frac{1}{e_2^c / \lambda T - 1}$$

where $c_1 8\pi hc$ and c_2=h c/k are universal constants.