UNIT - 3

APPLICATION OF DIFFERENTIAL CALCULUS

Formula :

S.No	y	$\frac{dy}{dx} = y'$
1.	c constant	0
2.	x^n	nx^{n-1}
3.	e^x	e^x
4.	e^{ax}	ae^{ax}
5.	$\sin x$	$\cos x$
6.	$\sin ax$	$\cos ax . a$
7.	$\cos x$	$-\sin x$
8.	$\cos ax$	$-\sin ax . a$
9.	$\tan x$	$\sec^2 x$
10	$\tan ax$	$\sec^2 ax . a$
11.	$\sec x$	$\sec x . \tan x$
12.	$\sec ax$	$\sec ax \tan ax . a$
13.	$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
14.	$\operatorname{cosec} ax$	$-\operatorname{cosec} ax \cot ax . a$
15.	$\cot x$	$-\operatorname{cosec}^2 x$
16.	$\cot ax$	$-\operatorname{cosec}^2 ax . a$
17.	$\log x$	$\frac{1}{x}$
18	$\log ax$	$\frac{a}{x}$
19	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
20	$\sin h x$	$\cos h x$

Product rule

$$d(uv) = uv' + vu'$$

Quotient rule

$$d\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

Defn : Curvature

The rate of bending of a curve at any point on it is called curvature of the curve at that point

Radius of curvature :

The reciprocal of the curvature of a curve at any point is called the radius of curvature It is denoted by ρ

Radius of curvature (Cartesian coordinate)

Let $y = f(x)$ be the given curve

then $$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{(1 + y_1^2)^{3/2}}{y_2}$$

If $\frac{dy}{dx} = \infty$, the radius of curvature is

$$\rho = \frac{\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{3/2}}{\frac{d^2x}{dy^2}}$$

21. $(\cosh x)' = \sinh x$
22. $(\tanh x)' = \operatorname{sech}^2 x$
23. $(\coth x)' = -\operatorname{cosech}^2 x$
24. $(\operatorname{cosech} x)' = -\operatorname{cosech} x \coth x$
25. $(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$

1. Find the curvature at any point on the curve

$x^2+y^2-6x-4y+10=0$

Circle $x^2+y^2+2gx+2fy+c=0$

Coeff x^2, y^2 must be 1

Centre $c=(-g,-f)$

Radius $r=\sqrt{g^2+f^2-c}$

The given curve, f - Principal focal point.
g - normal accelaration.

$2g=-6$ $\quad$ $2f=-4$

$g=-3$ $\quad$ $f=-2$

Centre, $c=(-g,-f)$
$=(3,2)$

Radius $r=\sqrt{g^2+f^2-c}$
$=\sqrt{9+4-10}$
$=\sqrt{3}$

Radius of curvature is $\rho=\sqrt{3}$

Curvature $\frac{1}{\rho}=\frac{1}{\sqrt{3}}$

Example 2 : Find the radius of curvature

$2x^2+2y^2+5x-2y+1=0$

Sol

Given $2x^2+2y^2+5x-2y+1=0$

$x^2+y^2+5/2x-y+1/2=0$

General form $x^2+y^2+2gx+2fy+c=0$

where $2g=5/2$ $\quad$ $2f=-1$ $\quad$ $c=1/2$

$g=5/4$ $\quad$ $f=-1/2$