

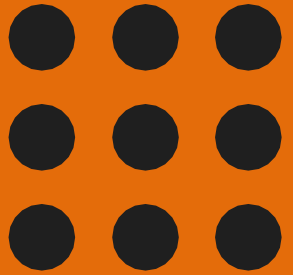


SNS COLLEGE OF TECHNOLOGY

Coimbatore-35

(An Autonomous Institution)

Accredited by NBA – AICTE and Accredited by NAAC – UGC with ‘A+’ Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai



DEPARTMENT OF MECHANICAL ENGINEERING

16ME401 FINITE ELEMENT ANALYSIS

IV YEAR VII SEM

UNIT V ISOPARAMETRIC FORMULATION

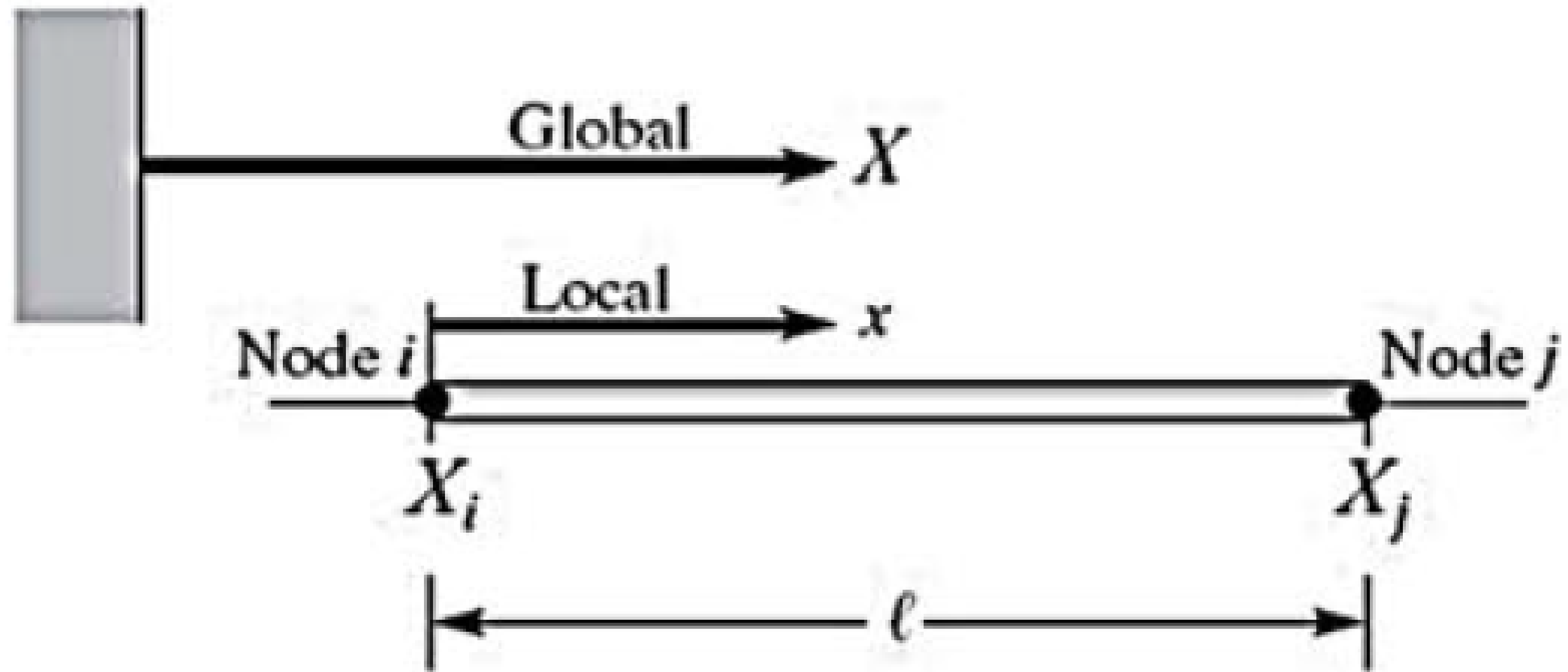
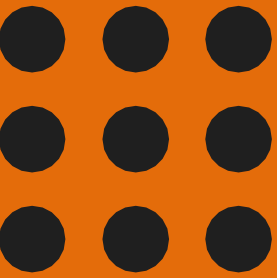
TOPIC – Isoparametric elements-Example Problem 1



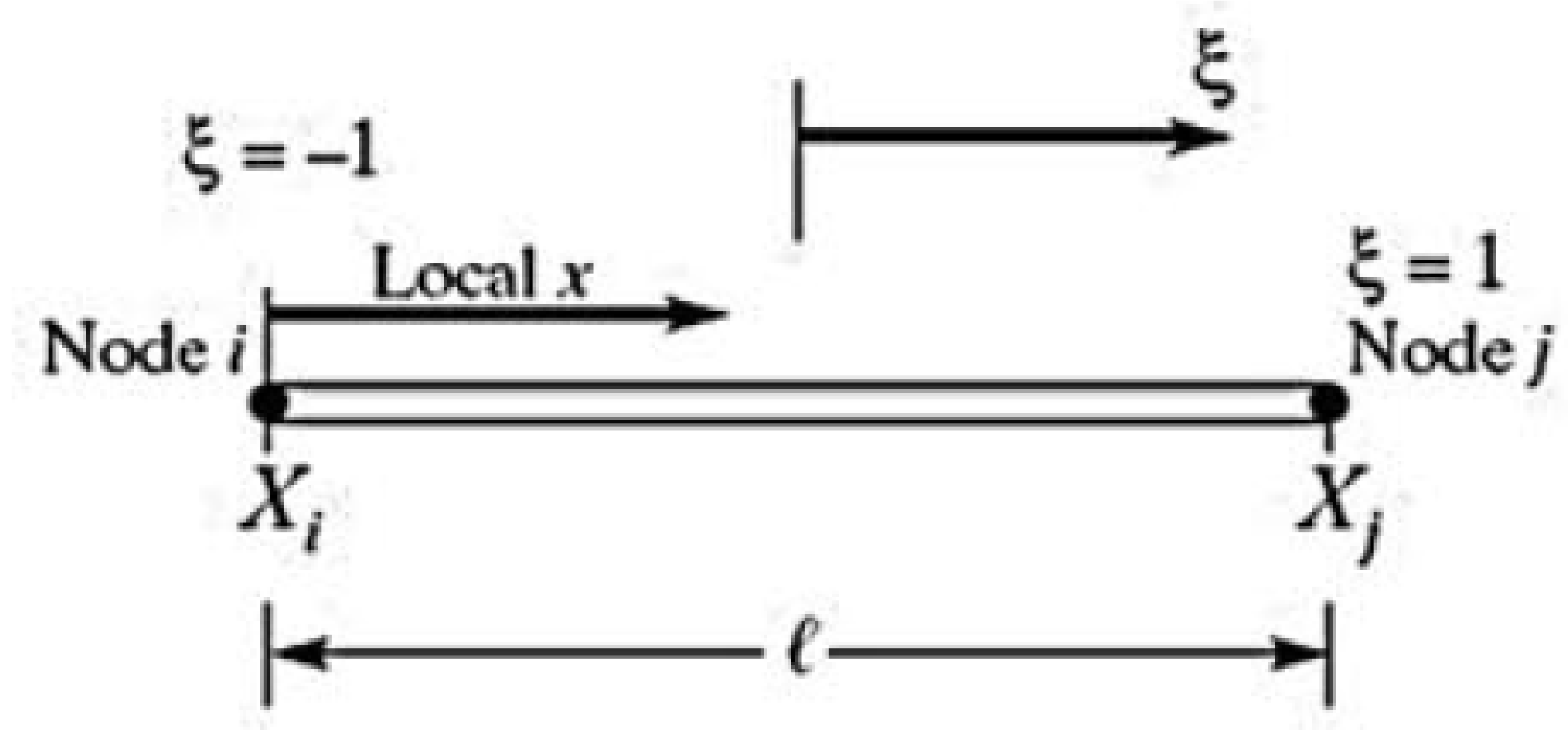
SNS *Design Thinkers*

Dr. M. SUBRAMANIAN, Professor & Mechanical

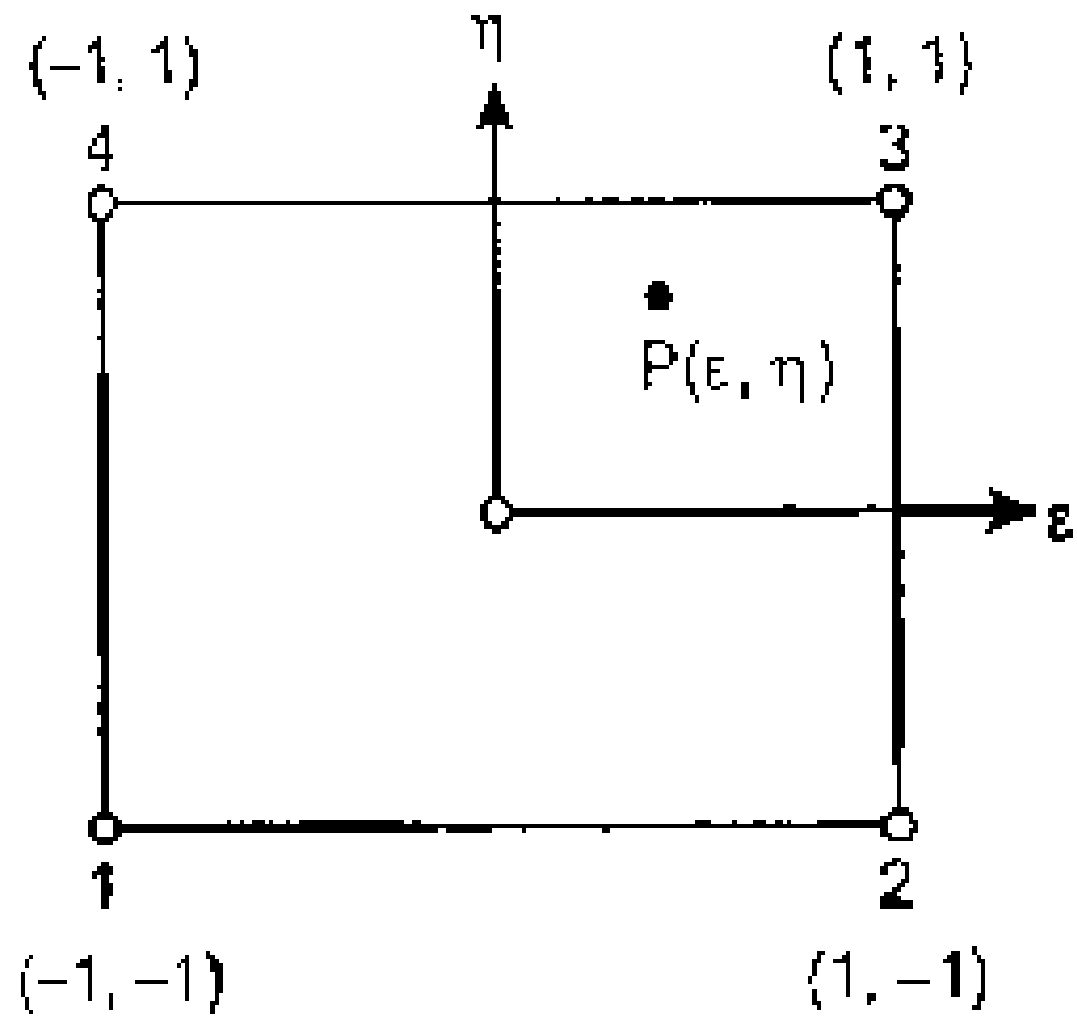
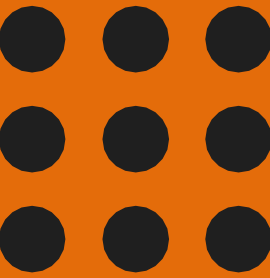




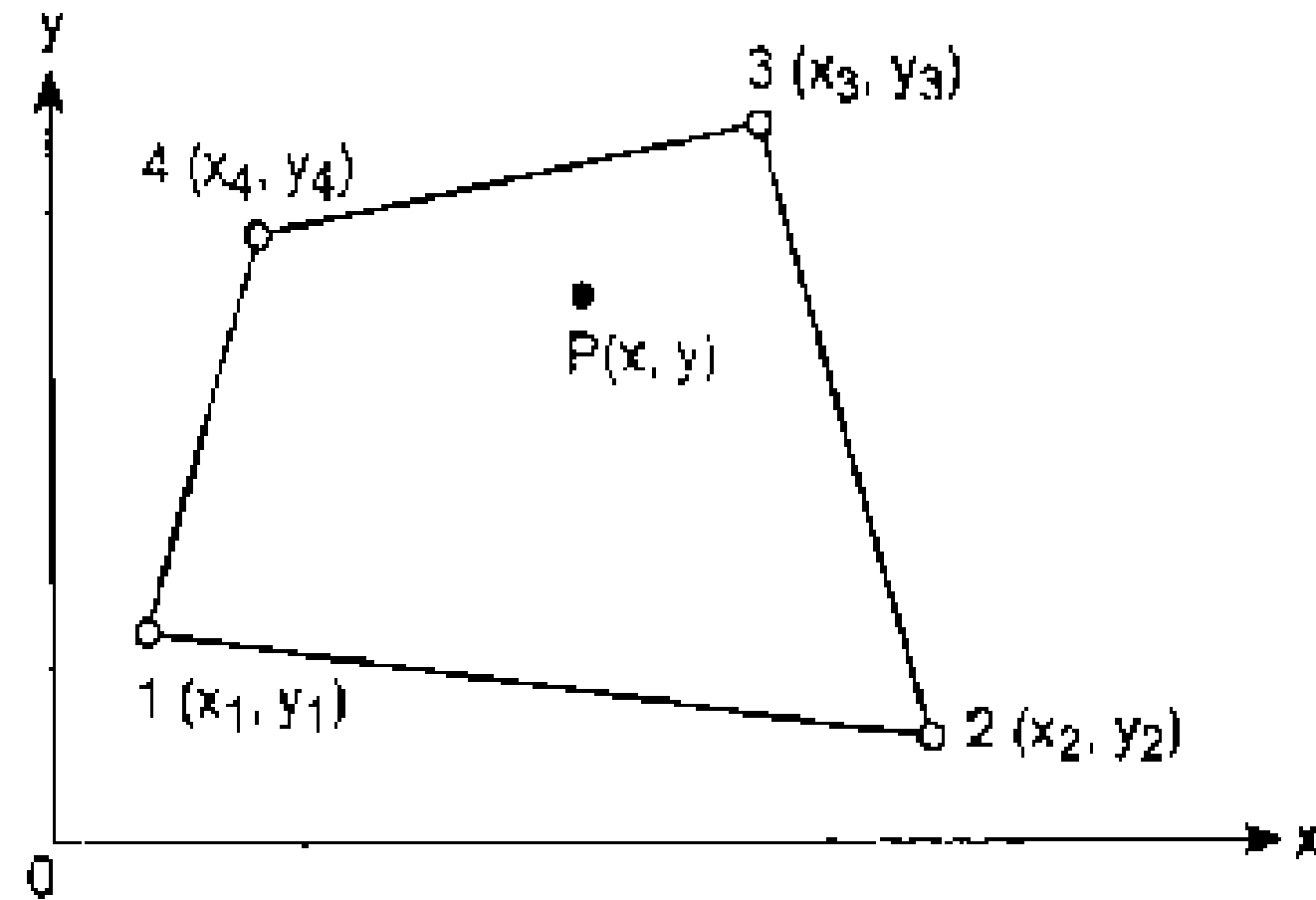
The relationship between a global coordinate X and a local coordinate x .



The relationship between the local coordinate x and the natural coordinate ξ .



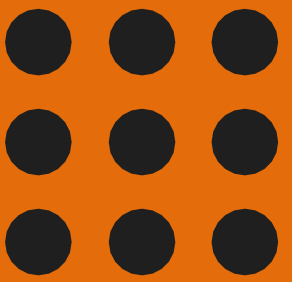
(a) Parent element



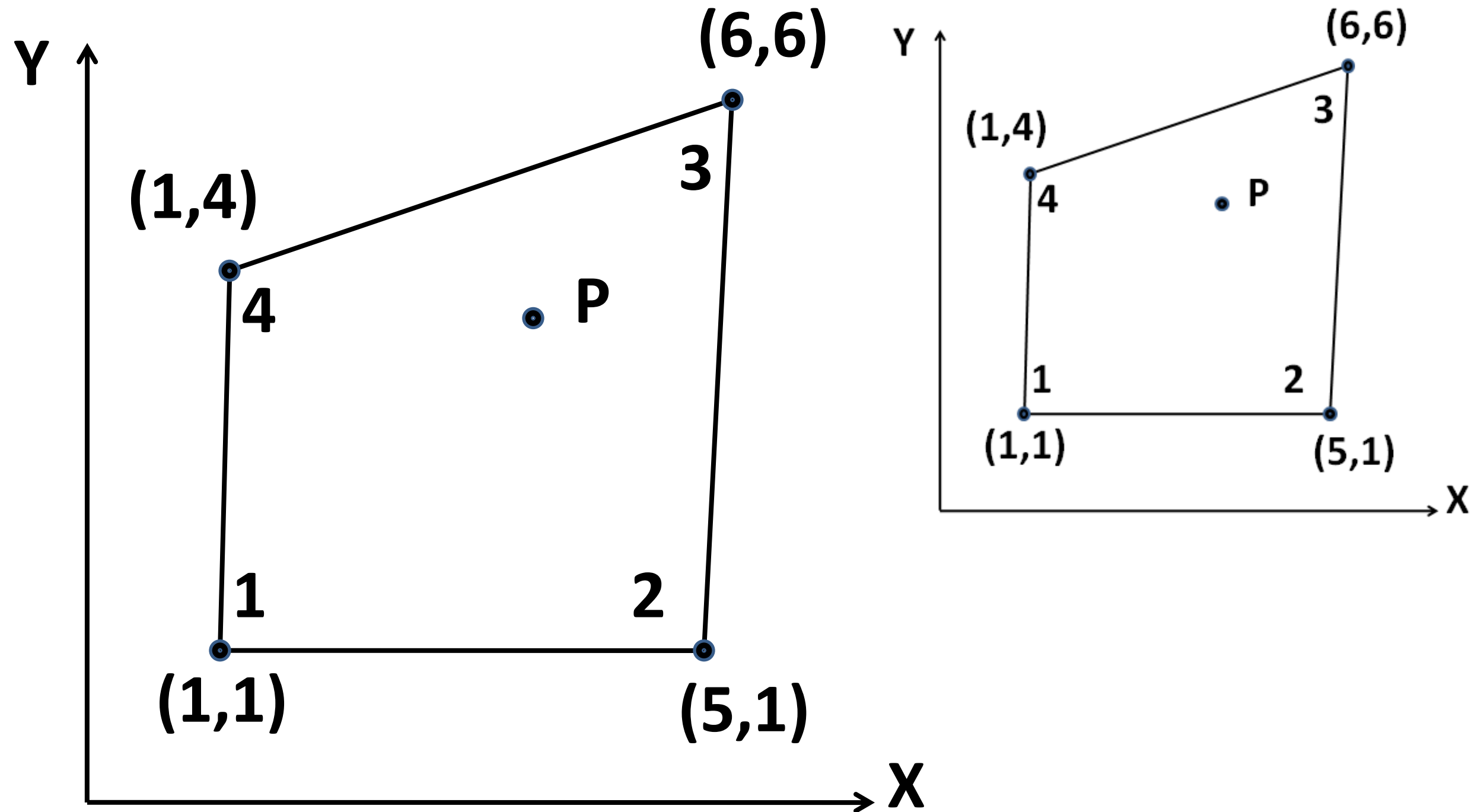
(b) Isoparametric quadrilateral element

Isoparametric Formulation





For the isoparametric four noded quadrilateral element shown in figure, determine the co-ordinates of point P which has local co ordinates $\xi=0.5$ and $\eta=0.5$





Solution:

Given data: Natural co-ordinates of point P

$$\varepsilon=0.5$$

$$\eta=0.5$$

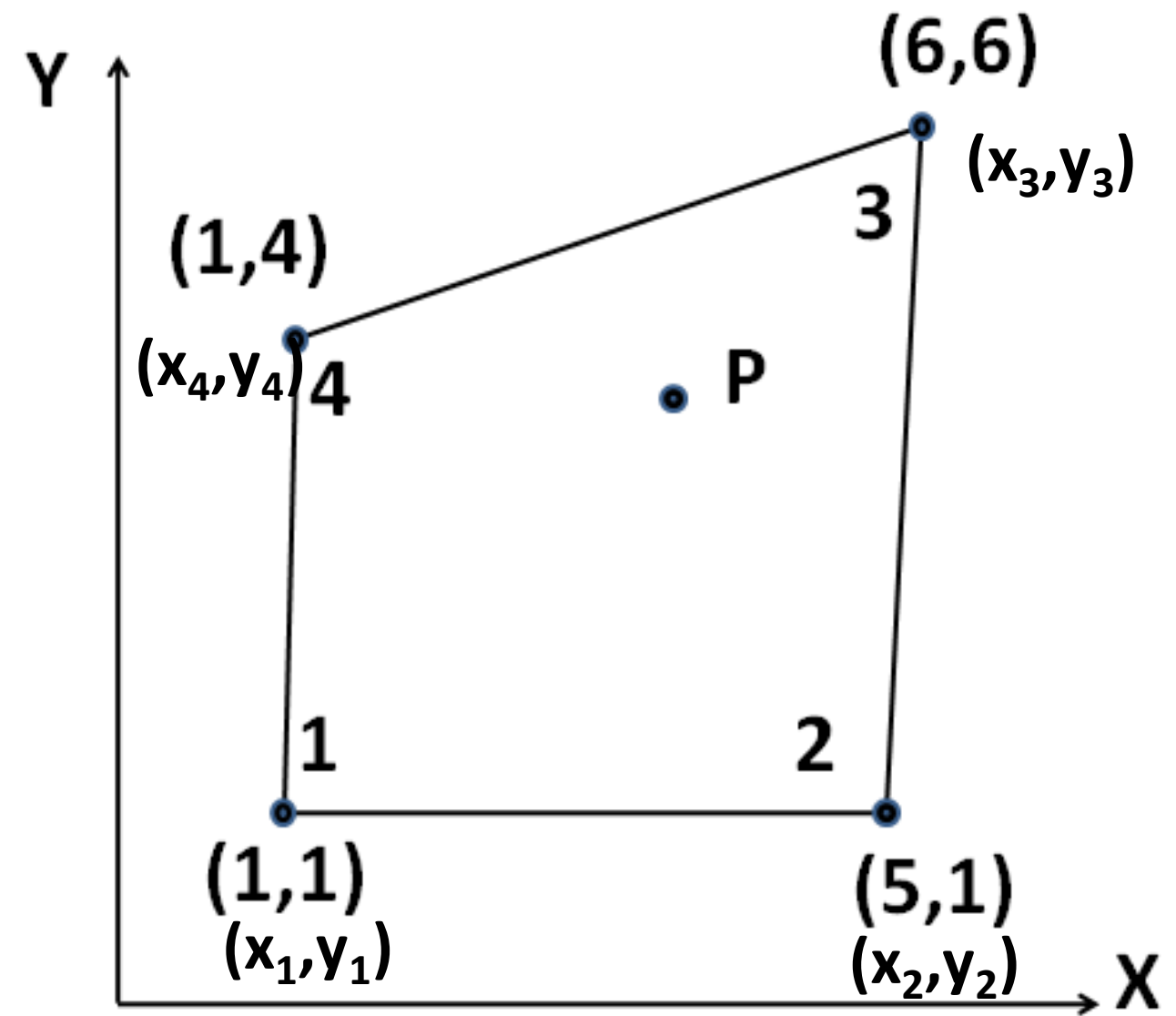
Cartesian co-ordinates of points 1, 2, 3 and 4,

$$x_1 = 1; \quad y_1 = 1$$

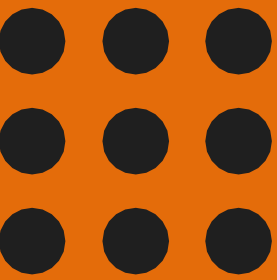
$$x_2 = 5; \quad y_2 = 1$$

$$x_3 = 6; \quad y_3 = 6$$

$$x_4 = 1; \quad y_4 = 4$$



To determine: Cartesian co-ordinates of the point P (x , y)





Shape functions for quadrilateral element are,

$$N_1 = \frac{1}{4}(1 - \varepsilon)(1 - \eta)$$

$$N_2 = \frac{1}{4}(1 + \varepsilon)(1 - \eta)$$

$$N_3 = \frac{1}{4}(1 + \varepsilon)(1 + \eta)$$

$$N_4 = \frac{1}{4}(1 - \varepsilon)(1 + \eta)$$

Substitute ε and η values in the above equations,

$$\Rightarrow N_1 = \frac{1}{4}(1 - 0.5)(1 - 0.5) = 0.0625$$

$$\Rightarrow N_2 = \frac{1}{4}(1 + 0.5)(1 - 0.5) = 0.1875$$

$$\Rightarrow N_3 = \frac{1}{4}(1 + 0.5)(1 + 0.5) = 0.5625$$

$$\Rightarrow N_4 = \frac{1}{4}(1 - 0.5)(1 + 0.5) = 0.1875$$

We know that,

$$\begin{aligned} \text{Co-ordinate, } x &= N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4 \\ &= 0.0625 \times 1 + 0.1875 \times 5 + 0.5625 \times 6 + 0.1875 \times 1 \end{aligned}$$

$$\boxed{x = 4.5625}$$

Similarly,

$$\begin{aligned} \text{Co-ordinate, } y &= N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4 \\ &= 0.0625 \times 1 + 0.1875 \times 1 + 0.5625 \times 6 + 0.1875 \times 4 \end{aligned}$$

$$\boxed{y = 4.375}$$

Result: The cartesian co-ordinates of the point P are (4.5625, 4.375).



Formula

$$N_1 = \frac{1}{4}(1 - \varepsilon)(1 - \eta),$$

$$N_2 = \frac{1}{4}(1 + \varepsilon)(1 - \eta)$$

$$N_3 = \frac{1}{4}(1 + \varepsilon)(1 + \eta),$$

$$N_4 = \frac{1}{4}(1 - \varepsilon)(1 + \eta)$$

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4$$

$$y = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4$$



Thank you