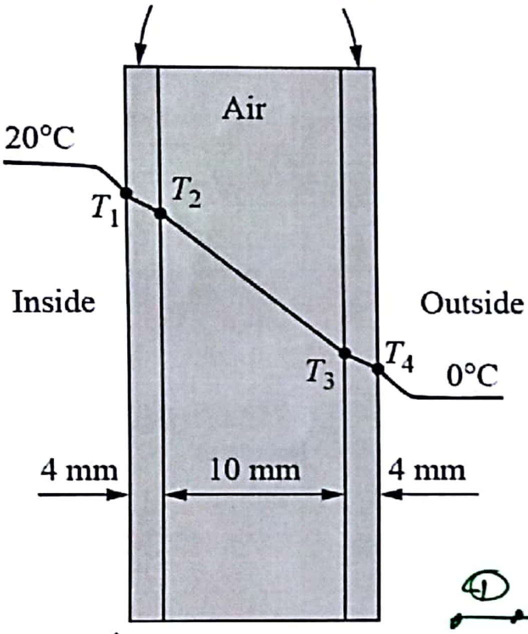
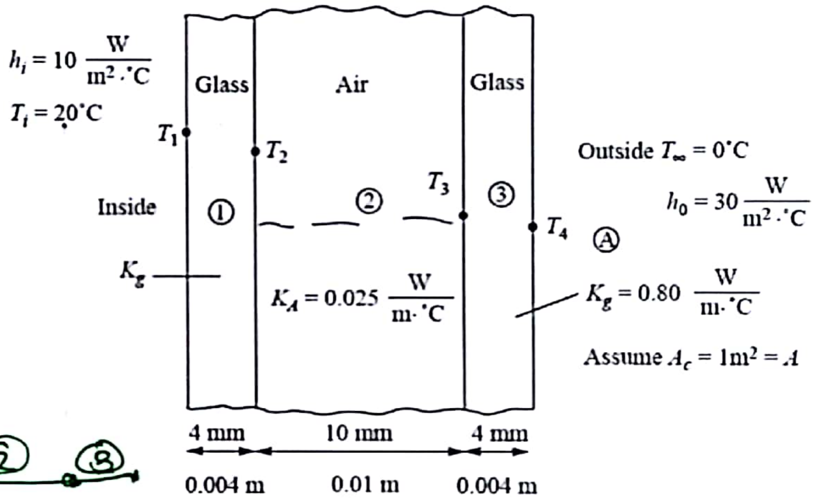
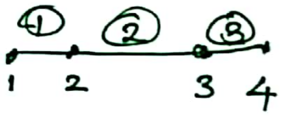
A double-pane glass window shown in Figure 0.0, consists of two 4-mm thick layers of glass with $k = 0.80 W/m°C$ separated by a 10 mm thick stagnant air space with $k = 0.025 W/m°C$. Determine (a) the temperature at both surfaces of the inside layer of glass and the temperature at the outside surfaces of glass, and (b) the steady rate of heat transfer in Watts through the double pane. Assume the inside room temperature $T_{i\infty} = 20°C$ with $h_i = 10 W/m^2°C$ and the outside temperature $T_{0\infty} = 0°C$ with $h_0 = 30 W/m^2°C$. Assume one-dimensional heat flow through the glass.







Stiffness matrix for Element ①

$$k^{(1)} = \frac{AK_1}{L_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + h_i A\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$$

From axial conduction   end convection at node i

$$= \frac{1 \times 0.8}{0.004}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + 10 \times 1\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} = \begin{bmatrix}200 & -200\\ -200 & 200\end{bmatrix} + \begin{bmatrix}10 & 0\\ 0 & 0\end{bmatrix}$$

$\frac{1}{200}$

$$= \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix}210 & -200\\ -200 & 200\end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array} \frac{W}{K}$$

Stiffness matrix for Element (2)

$$K^{(2)} = \frac{A_2 K_2}{L_2}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} = \frac{1\times0.025}{0.01}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} = \begin{array}{c|cc} & 2 & 3\\ \hline 2 & 2.5 & -2.5\\ 3 & -2.5 & 2.5\end{array}$$

Axial conduction

Stiffness matrix for Element (3)

$$K^{(3)} = \frac{A_3 K_3}{L_3}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} + h_o A\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

From axial conduction End convection at node j

$$= \frac{1\times0.8}{0.004}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} + 30\times1\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

200

$$= \begin{bmatrix}200 & -200\\-200 & 200\end{bmatrix} + \begin{bmatrix}0 & 0\\0 & 30\end{bmatrix} = \begin{array}{c|cc} & 3 & 4\\ \hline 3 & 200 & -200\\ 4 & -200 & 230\end{array}$$

The force vector

For Element ① $f^{(1)} = h_i T_\infty A\begin{bmatrix}1\\0\end{bmatrix}$ End convection at node i

$$= 10\times293\times1\begin{bmatrix}1\\0\end{bmatrix} = \begin{array}{c|c} 1 & 2930\\ 2 & 0\end{array}$$

For Element ② $f^{(2)} = \begin{bmatrix}0\\0\end{bmatrix}\begin{matrix}2\\3\end{matrix}$

For Element ③ $f^{(3)} = h_o T_\infty A\begin{bmatrix}0\\1\end{bmatrix}$ End convection at node j

$$= 30\times273\times1\begin{bmatrix}0\\1\end{bmatrix}$$

$$= \begin{bmatrix}0\\8190\end{bmatrix}_{4\times1}\begin{matrix}3\\4\end{matrix}$$

Assemble global equation. $[K][T] = [F]$

$$
\begin{array}{cccccc}
 & 1 & 2 & 3 & 4 & \\
\left[\begin{array}{c} \\ \\ \\ \\ \end{array}\right. & & & & & \\
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix} 210 & -200 & 0 & 0 \\ -200 & 200+2.5 & -2.5 & 0 \\ 0 & -2.5 & 2.5+200 & -200 \\ 0 & 0 & -200 & 230 \end{bmatrix}
\end{array}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} F_1 = 2930 \\ F_2 = 0 \\ F_3 = 0 \\ F_4 = 8190 \end{bmatrix}
$$

$$210 T_1 - 200 T_2 = 2930$$

$$-200 T_2 + 202.5 T_2 - 2.5 T_3 = 0$$

$$-2.5 T_2 \; T_2 + 202.5 T_3 - 200 T_4 = 0$$

$$-200 T_3 + 230 T_4 = 8190$$

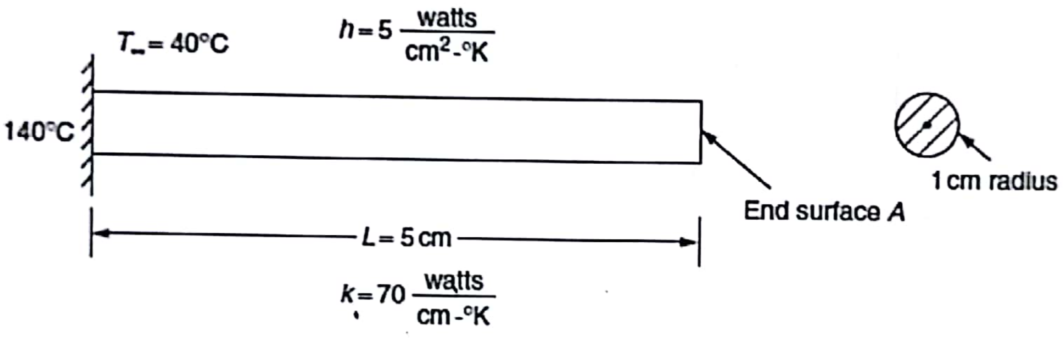
$T_1 = 289.3\,K = 16.3\,^\circ C$

$T_2 = 289.1\,K = 16.1\,^\circ C$

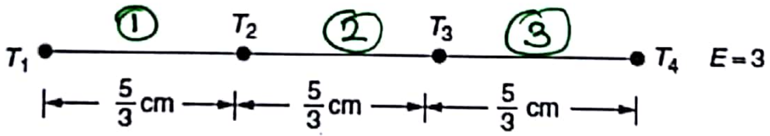
$T_3 = 279.4\,K = 1.4\,^\circ C$

$T_4 = 274.2\,K = 1.2\,^\circ C$

A fin is a one-dimensional heat transfer problem. One end of the fin is connected to a heat source (whose temperature is known $140°C$) and heat will be lost to the surroundings through the perimeter surface and the end. The temperature of the ambient air is $40°C$. The thermal conductivity of is $70(\frac{watts}{cm°K})$. The natural convective heat transfer coefficient associated with the surrounding air is $5\frac{watts}{cm^2°K}$. Find the temperature distribution in the fin shown in Figure 1.0 by including the effect of convection from the end surface A using three finite elements.



One dimensional rod



Finite element idealization

For elements (1) and (2) in the situation, the conductance and thermal load matrix are given by

$$[K]^{(e)} = \left\{ \frac{kA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hpl}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right]$$

From axial conduction perimeter convection.

$$[F]^{(e)} = \frac{hplT_f}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

perimeter convection.

Substituting for properties, we obtain.

$$[K]^{(1)} = \left\{ \frac{70 \times \pi (1)^2}{5/3} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{5 \times 2\pi \times 1 \times 5/3}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 132.05 & -132.05 \\ -132.05 & 132.05 \end{bmatrix} + \begin{bmatrix} 17.46702 & 8.733508 \\ 8.733508 & 17.46702 \end{bmatrix}$$

$$= \begin{array}{cc} \begin{array}{cc} \quad 1 & \qquad\quad 2 \end{array} & \\ \begin{bmatrix} 149.502 & -123.3165 \\ -123.3165 & 149.502 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

$$[K]^{(1)} = [K^{(2)}] = \begin{array}{cc} \begin{array}{cc} \quad 2 & \qquad\quad 3 \end{array} & \\ \begin{bmatrix} 149.502 & -123.3165 \\ -123.3165 & 149.502 \end{bmatrix} & \begin{array}{c} 2 \\ 3 \end{array} \end{array}$$

The thermal load matrix for element 1, 2

$$F^{(1)} = F^{(2)} = \frac{5 \times 2\pi \times 1 \times 5/3 \times 40}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1048.021 \\ 1048.021 \end{bmatrix} \begin{array}{c} 1 \\ 2 \end{array}$$

$$F^{(2)} = \begin{bmatrix} 1048.021 \\ 1048.021 \end{bmatrix} \begin{array}{c} 2 \\ 3 \end{array}$$

Including the boundary condition of the tip the conductance and load matrix for element (3) are obtain in the following manner.

$$K^{(e)} = \left\{ \underbrace{\frac{KA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{\text{From axial Conduction}} + \underbrace{\frac{hpl}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}_{\text{perimeter convection}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & hA \end{bmatrix}}_{\text{end convection at node } j} \right\}$$

$$= \begin{bmatrix} 149.502 & -123.3165 \\ -123.3165 & 149.502 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 5\times\pi(1)^2 \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{cc} 3 & 4 \end{array} \\ \begin{bmatrix} 149.502 & -123.3165 \\ -123.3165 & 165.222 \end{bmatrix} \end{array} \begin{array}{c} 3 \\ 4 \end{array}$$

load matrix F^3

$$= \frac{hpl T_f}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ hAT_f \end{bmatrix}$$

$$= \begin{bmatrix} 1048.21 \\ 1048.21 \end{bmatrix} + \begin{bmatrix} 0 \\ 628.8 \end{bmatrix}$$

$$= \begin{bmatrix} 1048.021 \\ 1676.821 \end{bmatrix} \begin{array}{c} 3 \\ 4 \end{array}$$

Assembly of the elements leads to the global conductance [K] and global load matrix (F)

$[K][T] = [F]$

$$\begin{bmatrix} 149.502 & -123.3165 & 0 & 0 \\ -123.3165 & \boxed{149.502 + 149.502} & -123.3165 & 0 \\ 0 & -123.3165 & \boxed{149.502 + 149.502} & -123.3165 \\ 0 & 0 & -123.3165 & 165.222 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 1048.022 \\ 2096.042 \\ 2096.042 \\ 1676.821 \end{bmatrix}$$

After incorporating the boundary condition $T_1 = 140°C$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -123.3165 & 299.004 & -123.3165 & 0 \\ 0 & -123.3165 & 299.004 & -123.3165 \\ 0 & 0 & -123.3165 & 165.222 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 140 \\ 2096.042 \\ 2096.042 \\ 1676.821 \end{bmatrix}$$

140°c

$$-123.3165[T_1] + 299.004[T_2] - 123.3165[T_3] = 2096.042$$

$$299.004\,T_2 - 123.3165\,T_3 = 2096.042 + (123.3165 \times 140)$$
$$= 19380.352 \quad \text{—(1)}$$

$$-123.3165\,T_2 + 299.004\,T_3 - 123.3165\,T_4 = 2096.042 \quad \text{—(2)}$$

$$-123.3165\,T_3 + 165.222\,T_4 = 1676.82 \quad \text{—(3)}$$

Solve above equation.

$T_1 = 140°$ $\quad T_2 = 94.68876°C$ $\quad T_3 = 72.5934°C$

$T_4 = 64.3303°C$