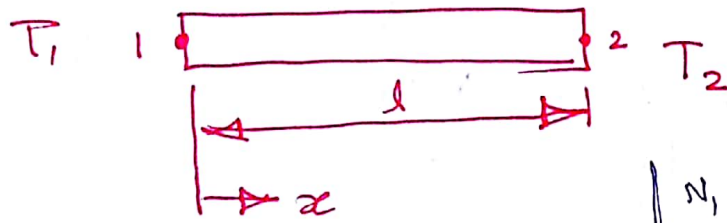


Shape function for one dimensional heat conduction problem.



$$T(x) = N_1 T_1 + N_2 T_2$$

$$N_1 = \frac{l-x}{l}$$

$$N_2 = \frac{x}{l}$$

$T(x)$ temperature function.

$$T(x) = a_0 + a_1 x$$

$$T = [1 \quad x] \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

At node 1 $T = T_1, x = 0$

At node 2 $T = T_2, x = l$

$$T_1 = a_0$$

$$T_2 = a_0 + a_1 l$$

$$\left. \begin{array}{l} T_1 = a_0 \\ T_2 = a_0 + a_1 l \end{array} \right\} \Rightarrow \text{Matrix form } \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & l \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & l \end{bmatrix}^{-1} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

$$= \frac{1}{l-0} \begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^{-1} = \frac{1}{(c_{11}c_{22} - c_{12}c_{21})} \begin{bmatrix} c_{22} & -c_{12} \\ -c_{21} & c_{11} \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{1}{l} \begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \rightarrow \text{Substitute in general eqn.}$$

$$T = [1 \ \alpha] \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$= [1 \ \alpha] \frac{1}{l} \begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

$$= \frac{1}{l} [l - \alpha \quad 0 + \alpha] \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{l - \alpha}{l} & \frac{\alpha}{l} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

$$= [N_1 \quad N_2] \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

Temperature function $T = N_1 T_1 + N_2 T_2$