Summary

Axisymmetric Triangular Element

- $u(r, z) = a_1 + a_2 r + a_3 z$

  $w(r, z) = a_4 + a_5 r + a_6 z$

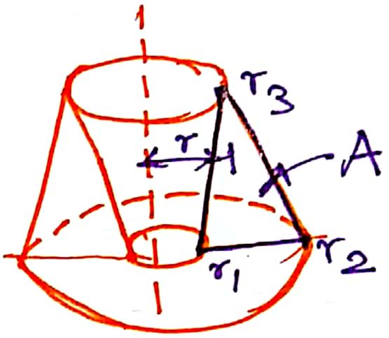
- $u_1 = a_1 + a_2 r_1 + a_3 z_1$

  $u_2 = a_1 + a_2 r_2 + a_3 z_2$

  $u_3 = a_1 + a_2 r_3 + a_3 z_3$

- Area of the triangle

$$A = \frac{1}{2}\begin{vmatrix} 1 & r_1 & z_1 \\ 1 & r_2 & z_2 \\ 1 & r_3 & z_3 \end{vmatrix}$$



- Shape function

$$= \left[\frac{\alpha_1 + \beta_1 r + \gamma_1 z}{2A} \quad \frac{\alpha_2 + \beta_2 r + \gamma_2 z}{2A} \quad \frac{\alpha_3 + \beta_3 r + \gamma_3 z}{2A} \right] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

- [D] - Stress-strain relationship matrix or Constitutive matrix

$$D = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} (1-\mu) & \mu & \mu & 0 \\ \mu & (1-\mu) & \mu & 0 \\ \mu & \mu & (1-\mu) & 0 \\ 0 & 0 & 0 & \left(\frac{1-2\mu}{2}\right) \end{bmatrix}$$

* $\{\sigma\} = [D]\{e\}$ → stress-strain. R.mukh

$\{\sigma\}$ ↳ Stress vector; $\{e\}$ → Strain Vector

$$\sigma = [D][B][S]$$

- $\{\epsilon\} = [B]\{\delta\}$

ϵ - Strain Vector $\begin{bmatrix} \epsilon_r \\ \epsilon_\theta \\ \epsilon_z \\ \gamma_{rz} \end{bmatrix}$

δ - Nodal displacement Vector $\begin{bmatrix} u_1 \\ w_1 \\ u_2 \\ w_2 \\ u_3 \\ w_3 \end{bmatrix}$

$[B]$ - Strain-displacement matrix (or) Gradient matrix

- $$[B] = \frac{1}{2A}\begin{bmatrix} B_1 & 0 & B_2 & 0 & B_3 & 0 \\ \frac{\alpha_1+\beta_1 r+\gamma_1 z}{r} & 0 & \frac{\alpha_2+\beta_2 r+\gamma_2 z}{r} & 0 & \frac{\alpha_3+\beta_3 r+\gamma_3 z}{r} & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & B_1 & \gamma_2 & B_2 & \gamma_3 & B_3 \end{bmatrix}$$

- $r = \frac{r_1 + r_2 + r_3}{3}$ $\quad Z = \frac{z_1 + z_2 + z_3}{3}$

- $\alpha_1 = r_2 z_3 - r_3 z_2$ | $\alpha_2 = r_3 z_1 - r_1 z_3$

$\beta_1 = z_2 - z_3$ | $\beta_2 = z_3 - z_1$

$\gamma_1 = r_3 - r_2$ | $\gamma_2 = r_1 - r_3$

$\alpha_3 = r_1 z_2 - r_2 z_1$

$\beta_3 = z_1 - z_2$

$\gamma_3 = r_2 - r_1$

- $[k] = 2\pi r A [B]^T [D][B]$