SNS COLLEGE OF TECHNOLOGY
Coimbatore-35
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DEPARTMENT OF MECHANICAL ENGINEERING

16ME401 Finite Element Analysis

Evaluate the stiffness matrix for the elements shown in Figure 0. The coordinates are given in units of millimeters. Assume plane stress conditions. Let E =210GPa, $\mu = 0.25$, and t = 10 mm.
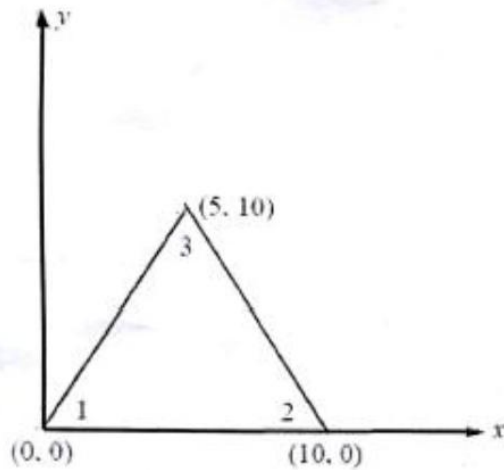
$(x_1, y_1) = (0,0)$

$(x_2, y_2) = (10, 0)$

$(x_3, y_3) = (5, 10)$

To Find :-

$$[k] = [B]^T [D] [B] \cdot A \cdot t$$

Step i) Area of triangular Element.

$$A = \frac{1}{2} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} = A = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 10 & 0 \\ 1 & 5 & 10 \end{bmatrix}$$

$$= \frac{1}{2} \left[1(10 \times 10 - 0) - 0 + 0 \right]$$

$$A = \frac{100}{2} = 50 \, mm^2$$

Step ii) Strain Displacement matrix

$$B = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

$$q_1 = y_2 - y_3 = 0 - 10 = -10$$

$$q_2 = y_3 - y_1 = 10 - 0 = 10$$

$$q_3 = y_1 - y_2 = 0 - 0 = 0$$

$$r_1 = x_3 - x_2 = 5 - 1 = -5$$

$$r_2 = x_1 - x_3 = 0 - 5 = -5$$

$$r_3 = x_2 - x_1 = 10 - 0 = 10$$

$$[B] = \frac{1}{2 \times 50} \times 5 \begin{bmatrix} 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -2 & -1 & 2 & 2 & 0 \end{bmatrix}$$

$$[B] = \frac{1}{20} \begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -2 & -1 & 2 & 2 & 0 \end{bmatrix}$$

$$[B]^T = \frac{1}{20} \begin{bmatrix} -2 & 0 & -1 \\ 0 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

Stress strain matrix [D] for plane stress condition:

$$[D] = \frac{E}{1 - \mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$$

$$[D] = \frac{2.1 \times 10^5}{1 - 0.25^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1 - 0.25}{2} \end{bmatrix}$$

$$[D] = \frac{2.1 \times 10^5}{1 - 0.25^2} [0.25] \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[D] = 56000 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[k] = [B]^T \, [D] \, [B] \cdot A \cdot t$$

$$= [D] \, [B]$$

$$= 56 \times 10^3 \times \frac{1}{20} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \times$$

$$\begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -2 & -1 & 2 & 2 & 0 \end{bmatrix}$$

$$= 2800 \begin{bmatrix} -8 & -1 & 8 & -1 & 0 & 2 \\ -2 & -4 & 2 & -4 & 0 & 8 \\ -1.5 & -3 & -1.5 & 3 & 3 & 0 \end{bmatrix}$$

To find :

$$[k] = [B]^T \, [D] \, [B] \cdot A \cdot t$$

$$= [B]^T \, [D] \, [B]$$

$$= \frac{2800}{2} \begin{bmatrix} -2 & 0 & -1 \\ 0 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} -8 & -1 & 8 & -1 & 0 & 2 \\ -2 & -4 & 2 & -4 & 0 & 8 \\ -1.5 & -3 & -1.5 & 3 & 3 & 0 \end{bmatrix}$$

$$= 70000 \begin{bmatrix} 17.5 & 5 & -14.5 & -1 & -3 & -4 \\ 5 & 10 & 1 & -2 & -6 & -8 \\ -14.5 & 1 & 17.5 & -5 & -3 & 4 \\ -1 & -2 & -5 & 10 & 6 & -8 \\ -3 & -6 & -3 & 6 & 6 & 0 \\ -4 & -8 & 4 & -8 & 0 & 16 \end{bmatrix}$$