

Problem :-

① A heat engine of 30% efficiency drives a heat pump of $COP = 5$. The heat is transferred both from engine and the heat pump to circulating water for heating building in winter. Find the ratio of heat transfer to the circulating water from the heat pump to the heat transfer to the circulating water from the heat engine.

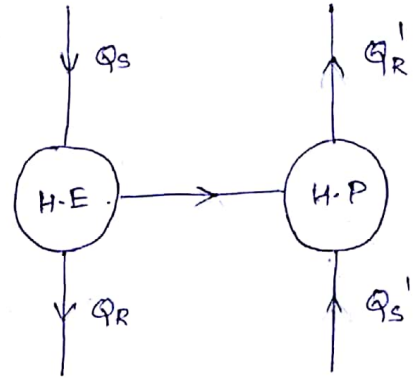
Given :-

$$\eta_{H.E} = 30\%$$

$$COP \text{ of H.P} = 5$$

To find :-

$$\frac{Q'_s}{Q_s}$$



Solution :-

Heat Engine $\eta_{H.E} = 0.3$

$$\frac{Q_s - Q_R}{Q_s} = 0.3$$

$$Q_s - Q_R = 0.3 Q_s$$

$$W = 0.3 Q_s$$

$$\boxed{Q_s = \frac{W}{0.3}} \quad \text{--- (1)}$$

For Heat pump,

$$COP = \frac{Q'_s}{Q'_s - Q'_R} = 5$$

$$Q'_s - Q'_R = \frac{Q'_s}{5}$$

$$Q'_s - \frac{Q'_s}{5} = Q'_R$$

$$Q'_s \left(1 - \frac{1}{5}\right) = Q'_R$$

$$0.8 Q'_s = Q'_R$$

$$\frac{Q'_R}{Q'_s} = 0.8 \Rightarrow Q'_R = 0.8 Q'_s$$

$$W = Q'_s - Q'_R = Q'_s - 0.8 Q'_s$$

$$W = 0.2 Q'_s$$

$$\boxed{Q'_s = \frac{W}{0.2}} \quad \text{--- (2)}$$

$$\frac{Q'_s}{Q_s} = \frac{W/0.2}{W/0.3} = \frac{0.3}{0.2} = 1.5$$

$$\boxed{\frac{Q'_s}{Q_s} = 1.5}$$

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② A reversible heat engine operating between reservoirs at 900 K and 300 K, drives a reversible refrigerator operating between reservoir at 300 K and 250 K. The heat engine receives 1800 kJ heat from 900 K reservoir. The net output from the combined engine refrigerator is 360 kJ. Find the heat transferred to the refrigerator and the net heat rejected to the reservoir at 300 K.

Given :-

$$T_1 = 900 \text{ K}$$

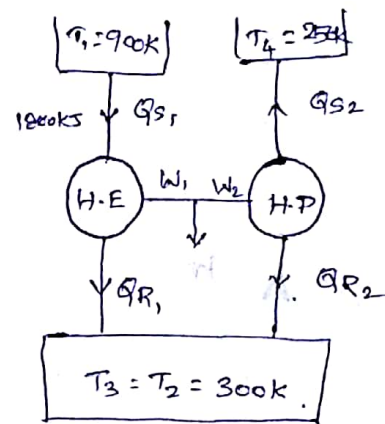
$$T_2 = 300 \text{ K}$$

$$T_3 = 250 \text{ K}, \quad W_1 - W_2 = 360 \text{ kJ}$$

To find :-

$$Q_{s2} = ?$$

$$Q_{R1} + Q_{s2} = ?$$



Solution :-

$$\eta_{\max} = \frac{T_H - T_L}{T_H} = \frac{900 - 300}{900} = 0.66 = 66\%$$

$$\eta_{\max} = \frac{W_1}{Q_{s1}}$$

$$W_1 = Q_{s1} \times \eta_{\max} = 1800 \times 0.66 = 1200 \text{ kJ}$$

$$W_1 = Q_{s1} - Q_{R1} \Rightarrow W_1 = 1800 - Q_{R1}$$

$$1200 = 1800 - Q_{R1}$$

$$\boxed{Q_{R1} = 600 \text{ kJ}}$$

$$\text{COP} = \frac{T_L}{T_H - T_L} = \frac{250}{300 - 250} = 5$$

$$\text{COP}_{\text{ref.}} = \frac{Q_{R2}}{Q_{s2} - Q_{R2}}$$

$$\frac{Q_{R2}}{W_2} = 5$$

$$W_1 - W_2 = 360 \text{ kJ}$$

$$-W_2 = 360 - W_1$$

$$W_2 = W_1 - 360$$

$$= 1200 - 360$$

$$\boxed{W_2 = 840 \text{ kJ}}$$

$$\frac{Q_{R2}}{840} = 5$$

$$Q_{R2} = 4200 \text{ kJ}$$

$$W_2 = Q_{S2} - Q_{R2}$$

$$840 = Q_{S2} - 4200$$

$$Q_{S2} = 840 + 4200$$

$$\boxed{Q_{S2} = 5040 \text{ kJ}}$$

$$\begin{aligned} \text{Net heat transfer at } 300 \text{ K} &= Q_{R1} + Q_{S2} \\ &= 600 + 5040 \\ &= 5640 \text{ kJ} \end{aligned}$$

② A Carnot heat engine cycle, works at maximum and minimum temperature of 1000°C and 40°C respectively. Calculate thermal efficiency and work done of $Q_S = 1010 \text{ kJ}$.

Given:-

$$T_H = 1000^\circ\text{C} = 1273 \text{ K}$$

$$T_L = 40^\circ\text{C} = 313 \text{ K}$$

$$Q_S = 1010 \text{ kJ}$$

To find:-

η, W

Solution:-

$$\eta = \frac{T_H - T_L}{T_H} = \frac{1273 - 313}{1273} = 0.754$$

$$\boxed{\eta = 75.4 \%}$$

$$\eta = \frac{W}{Q_S} \Rightarrow W = \eta \times Q_S$$

$$= 0.754 \times 1010$$

$$\boxed{W = 761.54 \text{ kJ}}$$

④ A Carnot engine receives heat from 600°C source. The efficiency ⑧ of the engine is 59%. Find the amount of heat supplied and heat rejected per kW of work output.

Given :-

$$\eta = 59\%$$

$$T_H = 600^{\circ}\text{C}$$

$$W = 1 \text{ kW}$$

To find :-

$$Q_S \text{ \& } Q_R$$

Solution :-

$$\eta = \frac{W}{Q_S} \Rightarrow 0.59 = \frac{1}{Q_S}$$

$$Q_S = \frac{1}{0.59}$$

$$Q_S = 1.694 \text{ kW}$$

$$W = Q_S - Q_R$$

$$Q_R = Q_S - W$$

$$= 1.694 - 1$$

$$Q_R = 0.694 \text{ kW}$$

⑤ The temperature in a domestic refrigerator is to be maintained at -10°C . The ambient air temperature is 30°C . If the heat leaving through the refrigerator is 3 kW. Determine the least power necessary to pump out this heat continuously.

Given :-

$$T_L = -10^{\circ}\text{C} = 263 \text{ K}$$

$$T_H = 30^{\circ}\text{C} = 303 \text{ K}$$

$$Q_S = 3 \text{ kW}$$

To find :-

Work.

Solution :-

$$\frac{Q_R}{T_L} = \frac{Q_S}{T_H} \Rightarrow Q_R = \frac{Q_S}{T_H} \times T_L = \frac{263}{303} \times 3$$

$$Q_R = 2.6 \text{ kW}$$

$$W = Q_S - Q_R = 3 - 2.6$$

$$W = 0.4 \text{ kW}$$

6) Two Carnot engines A and B are operated in series. The first one (A) receives heat at 870 K and rejects to a reservoir at temperature T. The second engine (B) receives the heat rejected by the first engine and in turn rejects to a heat reservoir at 300 K. Calculate the intermediate temperature T in °C between two heat engines for the following cases.

- The work output of the two engines are equal and.
- The efficiencies of the two engines are equal.

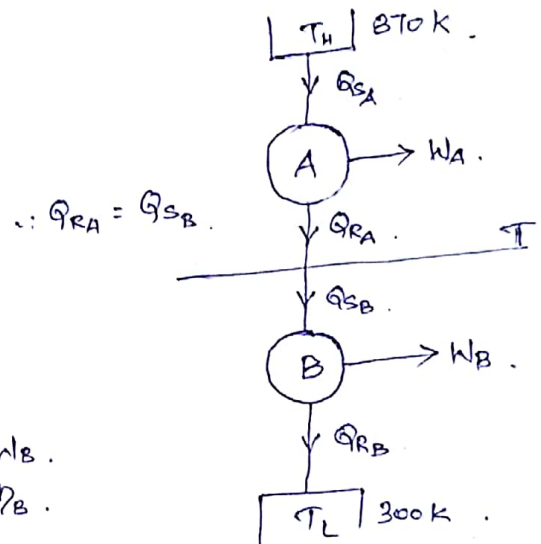
Given:-

$$T_H = 870 \text{ K.}$$

$$T_L = 300 \text{ K.}$$

$$a) W_A = W_B.$$

$$b) \eta_A = \eta_B.$$



To find :-

Intermediate Temperature T. @ $W_A = W_B$
 $\eta_A = \eta_B$.

Solution :-

Q_{SA} and Q_{RA} - Heat supplied and rejected to H.E - A.

Q_{SB} and Q_{RB} - Heat supplied and rejected to H.E - B.

$$a) W_A = W_B.$$

$$Q_{SA} - Q_{RA} = Q_{SB} - Q_{RB}.$$

$$\therefore Q_{RA} = Q_{SB}.$$

$$Q_{SA} - Q_{RA} = Q_{RA} - Q_{RB}.$$

$$Q_{SA} + Q_{RB} = 2Q_{RA}.$$

Q is proportional to T.

$$T_H + T_L = 2T$$

$$870 + 300 = 2T$$

$$T = \frac{1170}{2} = 585 \text{ K}$$

$$\boxed{T = 312^\circ \text{C}}$$

b) $\eta_A = \eta_B$.

$$\eta_A = \frac{W_A}{Q_{SA}} = \frac{Q_{SA} - Q_{RA}}{Q_{SA}} = \frac{870 - T}{870}$$

$$\eta_B = \frac{W_B}{Q_{SB}} = \frac{Q_{SB} - Q_{RB}}{Q_{SB}} = \frac{T - 300}{T}$$

$$\eta_A = \eta_B$$

$$\frac{870 - T}{870} = \frac{T - 300}{T}$$

$$T(870 - T) = 870(T - 300)$$

$$870T - T^2 = 870T - 261000$$

$$T^2 = 261000$$

$$T = 510.88 \text{ K}$$

$$T = 237.88^\circ\text{C}$$

Result :-

a) @ $W_A = W_B$, Intermediate temperature $T = 312^\circ\text{C}$.

b) @ $\eta_A = \eta_B$, Intermediate temperature $T = 237.88^\circ\text{C}$.