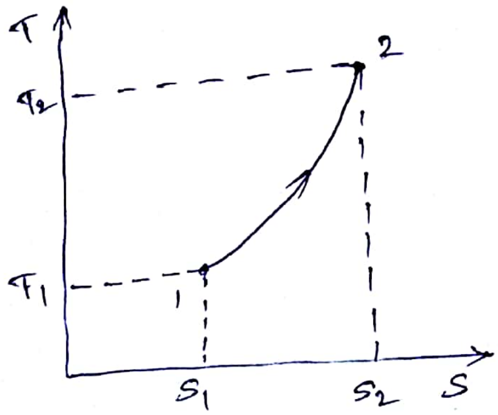
Entropy change of Ideal gas :-

Consider an ideal gas heated from state 1 and state 2 and they temperature increase from T_1 to T_2. During the heating process, they should be some change in entropy of the gas. Considered to the gas at absolute temperature T, change in entropy is given by $ds = \frac{dQ}{T}$.



In terms of temperature & volume

1^{st} law of thermodynamics.

$$Q = W + \Delta u$$

$$dQ = dW + du, \qquad \Rightarrow W = P\,dv, \quad du = mC_v\,dT.$$

$$dQ = P.dv + mC_v.dT \quad \text{—} \; ①$$

Dividing equation ① throughout by T

$$\frac{dQ}{T} = \frac{P.dv}{T} + mC_v\frac{dT}{T} \qquad PV = mRT, \quad P = \frac{mRT}{V}.$$

$$ds = \frac{mR\cancel{T}\,dv}{V.\cancel{T}} + mC_v.\frac{dT}{T}.$$

$$ds = mR\frac{dv}{V} + mC_v\frac{dT}{T}.$$

Integrating the above equation from state 1 to state 2

$$\int_1^2 ds = mR\int_1^2 \frac{dv}{V} + mC_v\int_1^2 \frac{dT}{T}.$$

$$[S]_1^2 = mR[\ln V]_1^2 + mC_v[\ln T]_1^2$$

$$= mR[\ln V_2 - \ln V_1] + mC_v[\ln T_2 - \ln T_1].$$

$$S_2 - S_1 = \boxed{ds = mR\ln\left(\frac{V_2}{V_1}\right) + mC_v\ln\left[\frac{T_2}{T_1}\right]} \quad \text{—} \; ②.$$

In terms of pressure & Temperature.

By, Gas equation. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}\times\frac{T_2}{T_1} \quad \text{—} \; ③.$$

Substituting ③ in ②. $\qquad \therefore R = C_p - C_v$

$$ds = mR\ln\left(\frac{P_1}{P_2}\times\frac{T_2}{T_1}\right) + mC_v\ln\left[\frac{T_2}{T_1}\right]$$

$$= mR\ln\left[\frac{P_1}{P_2}\right] + mR\ln\left[\frac{T_2}{T_1}\right] + mC_v\ln\left[\frac{T_2}{T_1}\right]$$

$$= mR\ln\left[\frac{P_1}{P_2}\right] + m[C_p - C_v]\ln\left[\frac{T_2}{T_1}\right] + mC_v\ln\left[\frac{T_2}{T_1}\right]$$

$$= mR\ln\left[\frac{P_1}{P_2}\right] + mC_p\ln\left[\frac{T_2}{T_1}\right] - \cancel{mC_v\ln\left[\frac{T_2}{T_1}\right]} + \cancel{mC_v\ln\left[\frac{T_2}{T_1}\right]}$$

$$\boxed{ds = mR\ln\left[\frac{P_1}{P_2}\right] + mC_p\ln\left[\frac{T_2}{T_1}\right]} \quad \text{—} \; ④$$

In terms of pressure & volume.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}.$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} \times \frac{V_2}{V_1}$$

Substituting this value in equation 2

$$ds = mR\ln\left[\frac{V_2}{V_1}\right] + mC_v\ln\left[\frac{P_2}{P_1} \times \frac{V_2}{V_1}\right]$$

$$= mR\ln\left[\frac{V_2}{V_1}\right] + mC_v\ln\left[\frac{P_2}{P_1}\right] + mC_v\ln\left[\frac{V_2}{V_1}\right]$$

$$= m(C_p - C_v)\ln\left[\frac{V_2}{V_1}\right] + mC_v\ln\left[\frac{P_2}{P_1}\right] + mC_v\ln\left[\frac{V_2}{V_1}\right]$$

$$= mC_p\ln\left[\frac{V_2}{V_1}\right] - \cancel{mC_v\ln\left[\frac{V_2}{V_1}\right]} + mC_v\ln\left[\frac{P_2}{P_1}\right] + \cancel{mC_v\ln\left[\frac{V_2}{V_1}\right]}.$$

$$= mC_p\ln\left[\frac{V_2}{V_1}\right] + mC_v\ln\left[\frac{P_2}{P_1}\right].$$