CLAUSIUS INEQUALITY :-

It states that when a system undergoes a cyclic process, the summation of $\frac{dQ}{T}$ around a closed cycle is less than or equal to zero.

$$\oint \frac{dQ}{T} \leq 0 .$$

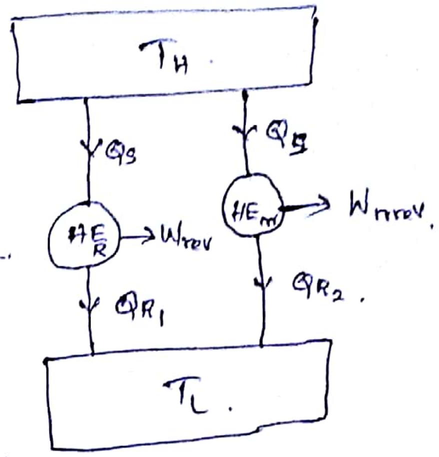
Consider an engine operating between two fixed temperature reservoirs T_H and T_L. let dQ_s, unit of heat be supplied at temperature T_H and dQ_R units of heat be rejected at temperature T_L during a cycle.

$$\text{Thermal efficiency } \eta = \frac{dQ_s - dQ_R}{dQ_s} .$$

$$\eta = \frac{T_H - T_L}{T_H} .$$

Mathematical relation.

$$\oint \frac{dq}{T} \leq 0 .$$



A reversible Carnot cycle,

$\eta_{rev} > \eta_{irrev}$. } both of them are working under same temperature.

Small temperature limit.

1 - Reversible engine.
1 - Irreversible heat engine.

} same as for heat pump. Max. COP get.

$$\oint_R \frac{dq}{T} = \int \frac{dq_s}{T_H} - \int \frac{dq_{R1}}{T_L}.$$

$$\Rightarrow \frac{Q_s}{T_H} - \frac{Q_{R1}}{T_L} = 0.$$

Reversible cycle, we have.

$$\frac{Q_s}{T_H} - \frac{Q_R}{T_L}$$

$\oint \frac{dq}{T} = 0.$ for a reversible cycle or heat engine.

Because of irreversible heat engine, work output is less as to rev. work output.

$$Q_{R_2} > Q_{R1}$$ ← always positive.

$$Q_{R_2} = Q_{R_1} + Q_{diff}.$$ >0

$$\oint_{irr} \frac{dq}{T} = \int \frac{dQ_s}{T_H} - \int \frac{dQ_{R2}}{T_L}$$

$$= \frac{Q_s}{T_H} - \left(\frac{Q_{R1}}{T_L} + \frac{Q_{diff}}{T_L} \right)$$

$$= \left(\frac{Q_s}{T_H} - \frac{Q_{R_1}}{T_L} \right) - \frac{Q_{diff.}}{T_L}$$

$$\oint_{irr} \frac{dq}{T} = - \frac{Q_{diff}}{T_L} < 0.$$

$\Rightarrow \boxed{\oint \frac{dq}{T} > 0}$

Solution :-

$$Q_2 = Q_1 \frac{T_2}{T_1} = 200 \times \frac{273}{373.15}$$

$$\boxed{Q_2 = 146.4\ J}$$

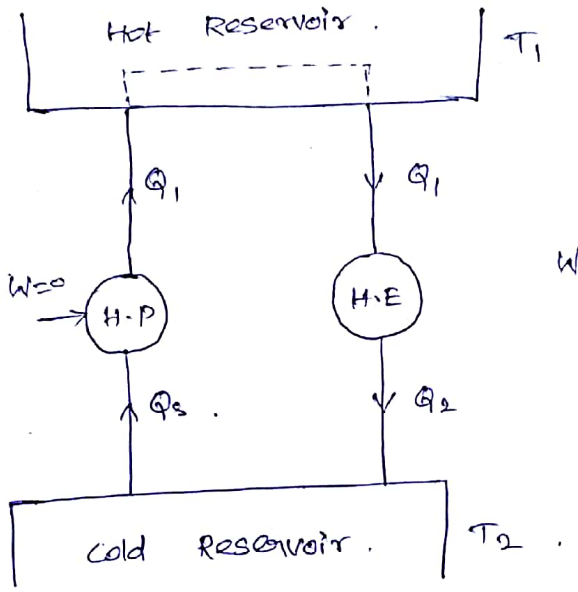
$$Work = Q_1 - Q_2 = 200 - 146.4 .$$

$$\boxed{W = 53.6\ J}$$

$$\eta = \frac{W}{Q_s} = \frac{53.6}{200} = 0.268 .$$

$$\boxed{\eta = 26.8\ \%}$$

Equivalence of Kelvin - Planck and Clausius Statement :-



$W_{net} = Q_1 - Q_2$.

⇒ First consider a cycle heat pump which transfer heat from a low temperature reservoir (T_2). To high temperature reservoir (T_1) with no other effect (or) work. It violates the clausius statement.

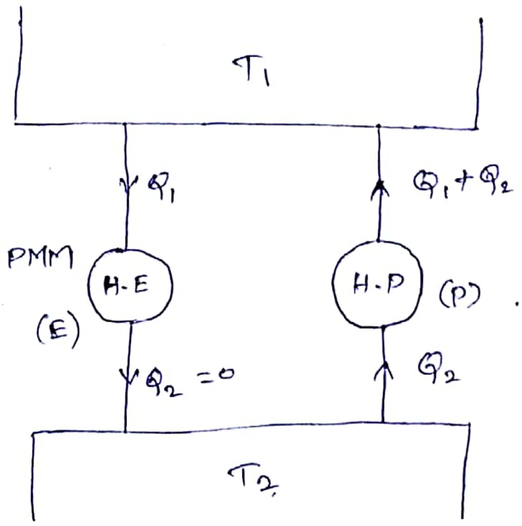
⇒ Let us assume a cyclic heat engine E operating between the same thermal energy reservoir, producing W_{net} in one cycle. The rate of working of the heat engine is such that it drawn an amount of heat Q_1 from the heat reservoir equal to that dischanged by the heat pump.

⇒ Then the hot reservoir may be eliminated and a the heat Q_1 dischanged by the heat pump is fed to the heat engine. So we see that the heat pump (P) and heat engine E acting together construct a heat engine operating in cycles. and producing.

net work while exchanging heat only with one body at a single fixed temperature (t_2). This violates the kelvin planck statement.

Let us consider a PMM of second kind (E). which produces net work in a cycle by exchanging heat with only one thermal energy reservoir (t_1). This violates the kelvin - planck statement.

Let us assume a cyclic heat pump (P) extracting heat Q_2 from low temperature reservoir t_2 and discharging heat to the high temperature reservoir at t_1 with expenditure of work W is equal to what the PMM_2 deliver in a complete cycle. So E & P together consitute a heat pump working in cycles and producing the sole effect of transferring heat from a lower to a higher temperature body. This violates the Clausius statement.



Problems :-

1) An inventor claims that his new engine will develop 3 kW for a heat addition of 240 kJ/min. The highest and the lowest temperature of the cycle are 1527°c and 327°c respectively. would you agree his claims ? use clausius inequality method.

Given :-

$$W = 3\ kW = 30 \times 60$$
$$= 1800\ kJ/min.$$
$$Q_1 = 240\ kJ/min.$$
$$T_1 = 1527°c = 1800\ K$$
$$T_2 = 327°c = 600\ K.$$

By clausius inequality,

$$\oint \frac{dQ}{T} = \frac{Q_s}{T_H} + \frac{Q_R}{T_L}$$

$$W = Q_s - Q_R.$$
$$1800 = 240 - Q_R.$$
$$Q_R = -1560\ kJ/min.$$

$$\frac{Q_s}{T_H} + \frac{Q_R}{T_L} = \frac{240}{1800} - \frac{1560}{600}$$
$$= -2.47\ kJ/min.$$

$$\oint \frac{dQ}{dT} < 0.$$

So, we agree his claim.