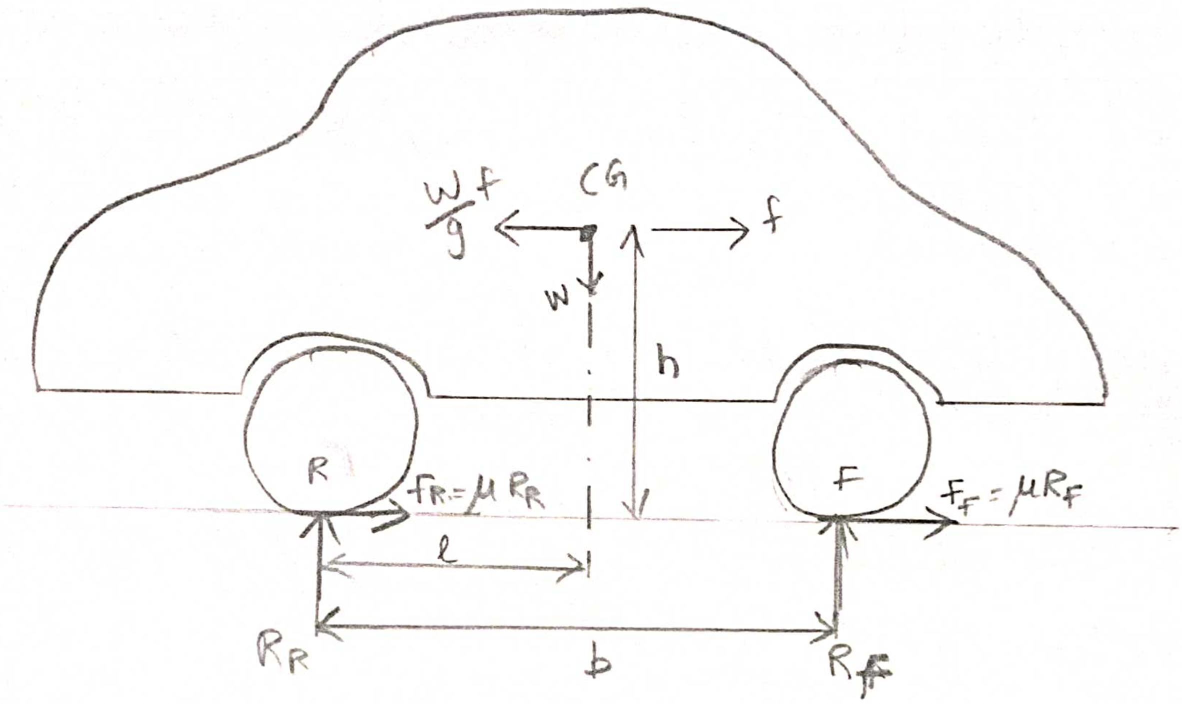
Calculation of maximum Acceleration, Maximum Tractive effort and Reaction force for Four Wheel drive.



The forces acting on the vehicle and giving rise to dynamic equilibrium are shown in the figure.

If

b = Wheelbase

h = height of CG from the road Surface

l = distance of CG from rear axle

μ = coefficient of adhesion between the tyres and the road Surface.

R_F & R_R = Total normal reaction at front and rear wheel

W = Weight of the Car.

Then the maximum tractive effort, $F_R = \mu R_R$, $F_F = \mu R_F$ produces maximum acceleration f and $(W/g) f$ is the inertia force opposite to acceleration

(i) With third differential

In this case both F_F and F_R comes into play. Assuming that limiting friction occurs at all the four wheels simultaneously, the maximum tractive effort

$$F = F_R + F_F$$

$$F = \mu R_R + \mu R_F \rightarrow (1)$$

$\Sigma V = 0$ gives $W = R_R + R_F \rightarrow (2)$

$\Sigma H = 0$ gives

$$(W/g) f = \mu R_R + \mu R_F = \mu (R_R + R_F)$$

Sub (2) in above equation.

$$(\cancel{W}/g) f = \mu \cancel{W}$$

$$\left(\frac{f}{g}\right) = \mu$$

(ii) With third differential

The torque at the front and rear wheels becomes equal with the application of third differential. Slip occurs at the wheels where the normal reaction is smaller and thus limits the tractive effort. Incase, the load distribution to the front and rear wheel is equal, the slip has to occur first at the front wheels because the static normal reaction at front wheels is reduced due to inertia effect.

Thus,

$$\Sigma V = 0 \text{ gives } W = R_R + R_F \rightarrow (3)$$

$$\Sigma H = 0 \text{ gives } \left(\frac{W}{g}\right) f = \mu R_R + \mu R_F \rightarrow (4)$$

and $\mu_s R_R = \mu R_F$ due to application of third differential, where μ_s is the critical working coefficient of friction being $< \mu$, the limiting value

Assuming slip to occur at front wheels first

$R_F < R_R$ then,

replace R_R with R_F in (4)

$$\frac{W}{g} f = \mu R_F + \mu R_F$$

$$2\mu R_F = (W/g) f \Rightarrow \boxed{R_F = \frac{W f}{2\mu g}}$$ (5)

$\Sigma M_R = 0$ gives

$$R_F b + (W/g) \cdot f h = W l.$$

Sub R_F in above equation

$$\frac{W f}{2\mu g} b + \frac{W}{g} f h = W l$$

$$\frac{W f}{g}\left[\frac{b}{2\mu} + h\right] = W l$$

$$\frac{f}{g}\left[\frac{b + 2\mu h}{2\mu}\right] = l$$

$$\boxed{\frac{f}{g} = \frac{2\mu l.}{b + 2\mu h}}$$

Sub (f/g) in (5)

$$R_F = \frac{W}{2\mu}\frac{f}{g}$$

$$= \frac{W}{\cancel{2\mu}}\left[\frac{\cancel{2\mu} l}{b+2\mu h}\right]$$

$$\boxed{R_F = \frac{Wl}{b+2\mu h}}$$

Sub R_F in (3)

$$W = R_F + R_R$$

$$W = \frac{Wl}{b+2\mu h} + R_R$$

$$R_R = W - \frac{Wl}{b+2\mu h}$$

$$R_R = W\left[1 - \frac{l}{b+2\mu h}\right]$$

$$\boxed{R_R = W\left[\frac{b+2\mu h - l}{b+2\mu h}\right]}$$

Assuming the slip to occur at rear wheel first, $R_R < R_F$

Replace R_F with R_R in (4)

$$\frac{W}{g}f = \mu R_R + \mu R_R.$$

$$2\mu R_R = \left(W/g\right) f$$

$$\boxed{R_R = \frac{W}{2\mu g} f} \rightarrow (6)$$

$\Sigma M_F = 0$ gives

$$R_R b = W(b-l) + W(f/g)h$$

Sub eqn (6) in above equation

$$\frac{W}{2\mu g} f \times b = W(b-l) + \frac{Wf}{g} h$$

$$\frac{W}{2\mu g} f b - \frac{Wfh}{g} = W(b-l)$$

$$\cancel{W} \frac{f}{g}\left[\frac{b}{2\mu} - h\right] = \cancel{W}(b-l)$$

$$\frac{f}{g}\left[\frac{b-2\mu h}{2\mu}\right] = b-l$$

$$\boxed{\frac{f}{g} = \frac{2\mu(b-l)}{b-2\mu h}}$$

Sub (f/g) in eqn (6)

$$R_R = \frac{W}{\cancel{2\mu}}\left[\frac{\cancel{2\mu}(b-l)}{b-2\mu h}\right]$$

$$R_R = W\left[\frac{b-l}{b-2\mu h}\right]$$

Sub R_R in 3

$$W = R_R + R_F$$

$$W = W\left[\frac{b-l}{b-2\mu h}\right] + R_F$$

$$R_F = W - W\left[\frac{b-l}{b-2\mu h}\right]$$

$$R_F = W\left[1 - \frac{b-l}{b-2\mu h}\right]$$

$$= W\left[\frac{\cancel{b}-2\mu h-\cancel{b}+l}{b-2\mu h}\right]$$

$$\boxed{R_F = W\left[\frac{l-2\mu h}{b-2\mu h}\right]}$$