SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
COIMBATORE-641 035

UNIT-III

PARTIAL DIFFERENTIAL EQUATIONS

PART- A

OBJECTIVE QUESTION AND ANSWERS

1. General Solution of $\frac{\partial^2 z}{\partial y^2} = 0$ is

   a) z = yf(x) + g(x) b) z = f(x)+g(x) c) None of the above

   Ans :(a)

2. General solution of $\frac{\partial z}{\partial x} = \sin x$ is

   a) z + cosx + f(y) b) z= sinx + f(y) c) z = -cosx + f(y)

   Ans :(c)

3. A solution of P.D.E which contains the maximum possible number of arbitrary constants is called a

   a) Particular Integral b) Complete Integral c) General Integral

   Ans :(b)

4. A solution obtained by giving particular values to the arbitrary constants in a complete integral is called

   a) General integral b) Complete Integral c) Particular integral

   Ans : (c)

5. A solution of a P.D.E which contains the maximum possible number of arbitrary functions is called

   Ans :(a)

   a) General integral b) Complete Integral c) Particular integral

6. Singular Integral of p+q = pq is

   Ans :(b)

   a) one Singular Integral b) two singular Integral c) no Singular integral

7. Complete integral of p = 2qx is

   Ans :(a)

   a) $z = ax^2 + ay + c$ b) $z = ax + ay + c$ c) $z = ax^2 + ay^2 + c$

8. A linear P.D.E with constant co- efficients in which all the partial derivatives are the same order is called

   a) homogeneous b) non homogeneous c) none of the above

   Ans :(a)

9. Particular integral of $(D^2+4DD')Z=e^x$ is
a) e^x b) e^{2x} c) e^{3x} Ans : (a)

10. General solution of $(D^3+DD'^2-D^2D'-D'^3)z=0$ is
a) $z = f_1(y+x) + f_2(y+ix) + f_3(y-ix)$ b) $z = f_1(y+x)$
c) $z = f_1(y+x) + f_2(y+x)$ Ans :(a)

11. The degree of the PDE $\frac{\partial^2 u}{\partial x \partial y}=\left(\frac{\partial u}{\partial z}\right)^3$ is
a) 3 b) 1 c) 0 Ans : (b)

12. Any PDE has a singular integral
a) True b) False c) None of the above Ans : (b)

13. Singular Integral of $z = px+ qy +pq$
a) $z = xy$ b) $z = x^2y$ c) $z = -xy$ Ans : (c)

14. The particular integral of $(D^2+3DD'+2D'^2)Z=\sin(x+5y)$ is
a) $-\frac{1}{66}\sin(x+5y)$ b) $\frac{1}{62}\sin(x+5y)$ c) None of the above Ans : (a)

15. Find the Particular Integral of $(D^2-4DD'+4D'^2)z=e^{2x+y}$ is
a) $\frac{x^2}{2}e^{2x+y}$ b) $\frac{x^3}{2}e^{2x+y}$ c) $\frac{x^2}{2}e^{2x-y}$ Ans :(a)

16. A PDE of the type $z = px + qy + f(p,q)$ is known as
a) Lagranges form b) Clairaut's type c) None of the above Ans : (b)

17. Find the Particular Integral of $(D^2+3DD'-4D'^2)z=x$ is a) $\frac{x^3}{6}$ b) $\frac{x^2}{5}$ c) $\frac{x^3}{7}$ Ans : (a)

18. Solve $(D^4-D'^4)z=0$ is

a) $z = f_1(y+x) + f_2(y-x)+f_3(y+ix) + f_4(y-ix)$ b) $z = f_1(y+x) + f_2(y-ix)$

c) $z=f_1(y - x)+x f_2(y - x)+x^2f_3(y- x) + x^3f_4(y-x)$

Ans :(a)

19. Find the general solution of $4\frac{\partial^2 z}{\partial y^2} - 12\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial^2 y} = 0$

a) $z = f_1\left(y + \frac{3x}{2}\right) + xf_2\left(y + \frac{3x}{2}\right)$ b) $z = f_1\left(y - \frac{3x}{2}\right) + xf_2\left(y - \frac{3x}{2}\right)$

Ans : (a)

c) None of the above

20. Write the differential equation corresponding to z = (x+a)(y+b) where a and b are arbitrary constants

Ans : (a)

a) $z = pq$ b) $z= p^2q$ c) $z = p^3q$

21. Find the Particular Integral of $(D^2 + DD')z = e^{x-y}$ is

Ans : (c)

a) $x\, e^{x+y}$ b)$x^2\, e^{x+y}$ c) $x\, e^{x-y}$

22. Find the Particular Integral of $(D^2 - DD' - 20D'^2)z = \sin(4x - y)$ is

Ans : (a)

a) $-\frac{x}{9}\cos(4x - y)$ b)-5cos(4x-y) c) None of the above

23. Complementary function of $(D^3 + D^2D' + DD'^2 + D'^3)Z=0$ is

a)$z=f_1(y - x)+x f_2(y - x)+x^2f_3(y- x)$ b) $z = f_1(y + x) + f_2(y + ix) + f_3(y-ix)$

Ans : (a)

c) $z = f_1(y + x)$

24. Form the p.d.e by eliminating 'f' from z= f(x+y)

a) p=q b) p=2q c) $p^2=q^2$

Ans : (a)

25. If the dependent variable and its partial derivative occur I the first degree, then we say that the p.d.e is

a) linear b)homogeneous c) non-homogeneous

Ans : (a)

26. Form a p.d.e by eliminating the arbitrary constants from z= ax+by

Ans : (a)

a) $z=px+qy$ b) $z= py + qx$ c) $z = p^2+q^2$

27. Form a p.d.e by eliminating the arbitrary constants from $z= a^2x+ay^2+b$

a)$4py^2=q^2$ b)$px=qy$ c) $py=qx$ Ans :(a)

28. Form a p.d.e by eliminating the arbitrary constants from $z= ax+by+a^2+b^2$

a)$z=px+qy+p^2+q^2$ b)$z=p^2x+q^2y+px+qy$ c) $z= px+qy+p+y$ Ans : (a)

29. Form a p.d.e by eliminating the arbitrary constants from $z= (x + a)^2+(y - b)^2$

a)$4z=p^2+q^2$ b) $z=p^2+q^2$ c)$z=p+q$ Ans : (a)

30. Form a p.d.e by eliminating the arbitrary constants from $z= (x^2 + a) +(y^2 + b)$

a)$pq=4xyz$ b)$z=4xy$ c) $pq=xyz$ Ans : (a)

31. Form a p.d.e by eliminating the arbitrary constants from $z= (2x^2 + a) +(3y -b)$

a)$pq=12xz$ b) $p=xyz$ c)$z=12xy$ Ans : (a)

32. Eliminate 'f' from $z= f(\sin x+\cos y)$

a) $p\sin y + q\cos x=0$ b) $p\cos y + p\sin x=0$ c) $p\sin x +q\cos y=0$ Ans : (a)

33. The complete integral of $\sqrt{p} + \sqrt{q} = 1$

a) $z = ax + (1 - \sqrt{a})^2 y + c$ b) $z=ax + (1- a)^2y+c$ c) $z=ax + by+c$ Ans :(a)

34. The complete integral of $\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$ is

a) $z = ax + by + (ab)^{\frac{3}{2}}$ b) $z=ax+by+cz$ c) $z= a^2x+b^2y+c^2z$ Ans :(a)

35. The complete integral of $\sqrt{p} + \sqrt{q} = x + y$ is

a) $\frac{(x+k)^3}{3} + \frac{(y-k)^3}{3} + b$ b) $\frac{(x+k)}{3} + \frac{(y-k)}{3} + b$ c) $\frac{x}{3} + \frac{y}{3} + b$ Ans : (a)

UNIT-III

PARTIAL DIFFERENTIAL EQUATIONS

PART-B

(TWO MARKS)

1. Find the PDE of all planes having equal intercepts on the x and y axis.

Ans: The equation of such planes is $\frac{x}{a}+\frac{y}{a}+\frac{z}{c}=1$ …… (1)

Partially differentiating (1) w.r.to x and y, we get

$\frac{1}{a}+\frac{p}{b}=0$ …… (2)

$\frac{1}{a}+\frac{q}{b}=0$ …… (3)

$$\left.\begin{aligned}(2)\Rightarrow \frac{p}{b}&=\frac{-1}{a}\\ (3)\Rightarrow \frac{q}{b}&=\frac{-1}{a}\end{aligned}\right\} \text{.....(4)}$$

From (4) , we get $\frac{p}{b}=\frac{q}{b}$

p = q

2. Form the PDE by eliminating a and b from $z=(x^2+a^2)(y^2+b^2)$

Ans: Given $z=(x^2+a^2)(y^2+b^2)$ …….. (1)

$p=\frac{\partial z}{\partial x}=2x(y^2+b^2)$ …..… (2)

$q=\frac{\partial z}{\partial y}=2y(x^2+a^2)$ …….. (3)

$(2)\Rightarrow y^2+b^2=\frac{p}{2x}$ …… (4)

$(3)\Rightarrow x^2+a^2=\frac{q}{2y}$ …… (5)

Sub (4) and (5) in (1), we get

$z=\frac{q}{2y}\frac{p}{2x}$

pq=4xyz

3. Eliminate the arbitrary function f from $z=f\left(\frac{xy}{z}\right)$ **and from PDE**

Ans: Given $z = f\left(\frac{xy}{z}\right)$

$$p = f'\left(\frac{xy}{z}\right)\frac{zy - xyp}{z^2} \quad \ldots\ldots\ldots (1)$$

$$q = f'\left(\frac{xy}{z}\right)\frac{zx - xyq}{z^2} \quad \ldots\ldots\ldots (2)$$

From (1) we get

$$f'\left(\frac{xy}{z}\right) = \frac{pz^2}{zy - xyp} \quad \ldots\ldots\ldots (3)$$

Sub (3) in (2) ,we get

$$q = \frac{pz^2}{zy - xyp}\frac{(zx - xyq)}{z^2}$$

4. Find the solution of $px^2 + qy^2 = z^2$

Ans: The S.E is $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$

Taking I two members ,we get

$$\frac{dx}{x^2} = \frac{dy}{y^2}$$

Integrating we get

$$\frac{-1}{x} = \frac{-1}{y} + c_1$$

$$u\left(\frac{1}{y} - \frac{1}{x}\right) = c_1$$

Taking last two members ,we get

$$\frac{dy}{y^2} = \frac{dz}{z^2}$$

Integrating we get

$$\frac{-1}{y} = \frac{-1}{z} + c_2$$

$$v\left(\frac{1}{z} - \frac{1}{y}\right) = c_2$$

The complete solution is

$$\phi\left(\frac{1}{y} - \frac{1}{x}, \frac{1}{z} - \frac{1}{y}\right) = 0$$

5. Find the singular integral of the partial differential equation $z = px + qy + p^2 - q^2$

Ans: The complete solution is $z = ax + by + a^2 - b^2$ …….. (1)

Now $\left.\begin{aligned} \frac{\partial z}{\partial a} = x + 2a = 0 \Rightarrow a = \frac{-x}{2} \\ \frac{\partial z}{\partial b} = y - 2b = 0 \Rightarrow b = \frac{y}{2} \end{aligned}\right\}$ .....(2)

Sub (2) in (1) , we get

$$z = \frac{-x^2}{2} + \frac{y^2}{2} + \frac{x^2}{4} - \frac{y^2}{4}$$

$$= \frac{-x^2}{4} + \frac{y^2}{4}$$

$y^2 - x^2 = 4z$ which is S.I

6.Form the PDE by eliminating the arbitrary constants from $z = (x-a)^2 + (y-b)^2 + 1$

Ans: $z = (x-a)^2 + (y-b)^2 + 1$ ……. (1)

$$p = \frac{\partial z}{\partial x} = 2(x-a) \quad \text{…… (2)}$$

$$q = \frac{\partial z}{\partial y} = 2(y-b) \quad \text{…… (3)}$$

Sub (2) and (3) in (1) we get

$$z = \frac{p^2}{4} + \frac{q^2}{4} + 1$$

7.Write the complete solution of p+q=x+y

Ans: Let p+q=x+y=k

p-x=k , y-q=k

p=k+x q=y-k

$$z = \int p\,dx + \int q\,dy$$

$$z = \int (k+x)\,dx + \int (y-k)\,dy$$

$$= \frac{(k+x)^2}{2} + \frac{(y-k)^2}{2} + c$$

8.Find the partial integral of $(D^2 - 2DD' + D'^2)z = e^{x-y}$

Ans: $$P.I = \frac{1}{D^2 - 2DD' + D'^2} e^{x-y}$$

$$= \frac{1}{4} e^{x-y}$$

9. **Find the complete integral of q=2px**

Ans: q=2px

$$xp = \frac{q}{2} = k$$

$$p = \frac{k}{x}, \qquad q = 2k$$

$$z = \int p\,dx + \int q\,dy$$

$$z = k\log x + 2ky + c$$

10.Solve $(D^3 - 3DD'^2 + 2D'^3)z = 0$

Ans: A.E is $(m^2 - 3m + 2) = 0$

(m-1)(m-1)(m+2)=0

m=1,1,-2

$$z = f_1(y + x) + xf_2(y + x) + f_3(y - 2x)$$

11.Find the complete solution of the PDE $\sqrt{p} + \sqrt{q} = 1$

Ans: Given $\sqrt{p} + \sqrt{q} = 1$ ……. (1)

Let z = ax+by+c …… (2) be the solution of (1)

$$\left.\begin{aligned} \frac{\partial z}{\partial x} = p = a \\ \frac{\partial z}{\partial y} = q = b \end{aligned}\right\} \text{.....(3)}$$

Sub (3) in (1)

$\sqrt{a} + \sqrt{b} = 1$ ……. (4)

z = ax+by+c is a solution of (1)

From (4) $\sqrt{a} = 1 - \sqrt{b}$ ……(5)

Sub (5) in (2)

$z = (1 - \sqrt{b})^2 x + by + c$ which is complete integral

12. Find the Complete integral of the partial differential equation $z = px + qy + p^2 + q^2$

Ans: Sub p = a and q = b

$$z = ax + by + a^2 + b^2$$

13. Eliminate the arbitrary function f from $z = f\left(\frac{x}{y}\right)$ and from PDE

Ans: Given $z = f\left(\frac{x}{y}\right)$

$$p = f'\left(\frac{x}{y}\right)\frac{1}{y} \quad \ldots\ldots\ldots (1)$$

$$q = f'\left(\frac{x}{y}\right)\frac{-x}{y^2} \quad \ldots\ldots\ldots (2)$$

Sub (1) in (2) ,we get

$$q = py\left(\frac{-x}{y^2}\right)$$

$$px + qy = 0$$

14.Solve $(D^2 - 3DD' + 2D'^2)z = 0$

Ans: A.E is $m^2 - 3m + 2 = 0$

m=1,2

$$z = f_1(y + x) + f_2(y + 2x)$$

15.Find the PDE by eliminating the arbitrary constants in $z = a(x + y) + b$

Ans: $z = a(x + y) + b$ ….. (1)

Partially differentiating (1) w.r.to 'x' and 'y' we get

$$\frac{\partial z}{\partial x} = p = a \quad \ldots\ldots (2)$$

$$\frac{\partial z}{\partial y} = q = a \quad \ldots\ldots (3)$$

From (2) and (3) , p = q

16. Solve $(D^3 - 2D^2D')z = 0$

Ans: A.E is $m^3 - 2m^2 = 0$

m=0,0,2

$$z = f_1(y) + xf_2(y) + f_3(y + 2x)$$

17.Find the complete solution of pq=xy

Ans: $pq=xy$

$$\frac{p}{x}=\frac{y}{q}$$

Let $\frac{p}{x}=\frac{y}{q}=k$

$$p=kx\ ,q=\frac{y}{k}$$

$$z=\int p\,dx+\int q\,dy$$

$$=\int kx\,dx+\int\frac{y}{k}dy$$

$$=k\frac{x^2}{2}+\frac{1}{k}\frac{y^2}{2}+c$$

18. Solve $\frac{\partial^2 z}{\partial x^2}-\frac{4\partial^2 z}{\partial x\,\partial y}+\frac{4\partial^2 z}{\partial y^2}=0$

Ans: A.E is $m^2-4m+4=0$

$$(m-2)^2=0$$

m=2,2

$$z=f_1(y+2x)+2f_2(y+2x)$$

20. Write Particular integral of $\frac{\partial^2 z}{\partial x^2}-\frac{5\partial^2 z}{\partial x\,\partial y}+\frac{6\partial^2 z}{\partial y^2}=e^{x+y}$

Ans: $P.I=\frac{1}{D^2-5DD'+6D'^2}e^{x+y}$ Sub $D=1$ and $D'=1$

$$=\frac{1}{1-5+6}e^{x+y}$$

$$=\frac{1}{2}e^{x+y}$$

21. Write Particular integral of $(D^2+3DD'+2D'^2)z=\sin(x+5y)$

Ans: $P.I=\frac{1}{D^2+3DD'+2D'^2}\sin(x+5y)$ Sub $D^2=-1$, $DD'=-5$ and $D'^2=-25$

$$=\frac{1}{-1-15-50}\sin(x+5y)$$

$$=\frac{1}{-66}\sin(x+5y)$$

22.Find the singular Integral of $z = px+qy+p^2$

Ans: Complete solution is $z = ax+by+a^2$ …….. (1)

$$\frac{\partial z}{\partial a}=x+2a=0$$ …….. (2)

$$\frac{\partial z}{\partial b}=y=0$$ …….. (3)

From (2) we get

x= - 2a (or) a= - x/2 ……. (4)

Sub (3) and (4) in (1) , we get

$$z = \frac{-x^2}{2} + \frac{x^2}{4} = \frac{-x^2}{4}$$

$4z = -x^2$

23. **Write Particular integral of** $(D^2 - DD')z = \sin(x+y)$

Ans:

$$P.I = \frac{1}{D^2 - DD'}\sin(x+y) \quad \text{Sub} \quad D^2 = -1 \,,\, DD' = -1$$

$$= \frac{1}{-1+1}\sin(x+y)$$

$$= \frac{x}{2D}\sin(x+y)$$

$$= \frac{-x}{2}\cos(x+y)$$

24. **Write Particular integral of** $(D^2 + 2DD' + D'^2)z = e^{(x-y)}$

Ans:

$$P.I = \frac{1}{D^2 + 2DD' + D'^2}e^{(x-y)} \quad \text{Sub} \quad D = 1 \,,\, D' = -1$$

$$= \frac{1}{1-2+1}e^{(x-y)}$$

$$= \frac{x}{2D + 2D'}e^{(x-y)}$$

$$= \frac{x^2}{2}e^{(x-y)}$$

25.Find the complete solution of p-q=0

Ans: p-q=0

This is of the type F(p,q)=0 …… (1)

Let z=ax+by+c ….. (2) be the solution of PDE

From (2) we get p=a , q=b ……. (3)

Sub (3) in (1), we get

a-b=0 $\Rightarrow$ a=b ……. (4)

Sub (4) in (2) , we get z= a(x+y)+c

UNIT-III

PARTIAL DIFFERENTIAL EQUATIONS

PART-C

1.		Solve $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$	
2.		Solve $(D^2+DD'-6D'^2)z=y\cos x$	
3.		Solve $z=px+qy+\sqrt{p^2+q^2+1}$	
4.		Solve $(D^3-7DD'^2-6D'^3)z=\sin(2x+y)$	
5.		Find the singular integral of $z=px+qy+p^2+pq+q^2$	
6.		Solve the PDE $(x-2z)p+(2z-y)q=y-x$	
7.		Solve $(D^2+3DD'-4D'^2)z=\cos(2x+y)+xy$	
8.		Solve $(D^2-DD'+2D)z=e^{2x+y}+4$	
9.		Solve $(mz-ny)p+(nx-lz)q=ly-mx$	
10.		Solve $(D^3+D^2D'-DD'^2-D'^3)z=e^x\cos 2y$	
11.		Solve $p(1+q)=qz$	
12.		Solve $(D^2-DD'-30D'^2)z=xy+e^{2x+y}$	
13.		Solve $(3z-4y)p+(4x-2z)q=2y-3x$	
14.		From the PDE by eliminating the arbitrary function f & g from $z=f(x+ct)+g(x-ct)$	
15.		Solve $z=1+p^2+q^2$	
16.		From the PDE by eliminating the arbitrary function ϕ from $\phi(x^2+y^2+z^2, ax+by+cz)=0$	
17.		Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$	
18.		Solve $(D^3+D^2D'-4DD'^2-4D'^3)z=\cos(2x+y)$	
19.		Solve $(2D^2-DD'-D'^2+6D+3D')z=xe^y$	
20.		Solve $(x-y)p+(y-x-z)q=z$	
21		Solve $(1+y)p+(1+x)q=z$	
22.		Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$	

23.		Solve $(D^2 + 4DD' - 5D'^2)z = x + y^2 + \pi$	
24.		Solve $(D^2 - 6DD' + 5D'^2)z = e^x \sinh y + xy$	
25.		Solve $(D^3 - 4D^2D' + 4DD'^2)z = 6\sin(3x + 6y)$	
26.		Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = \cos 2x \cos y$	
27.		Solve $(D^2 + 4DD' - 5D'^2)z = e^{2x-y} + \sin(x - 2y)$	
28.		Solve $(D^2 - 2DD' + D'^2)z = x^2y^2e^{x+y}$	