



SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB301 – ANALOG AND DIGITAL COMMUNICATION

III B.E. ECE₁ / V SEMESTER

UNIT 5 – INFORMATION THEORY AND ERROR CONTROL CODING

TOPIC – ERROR CONTROL CODING



ERROR CONTROL CODING



Purpose

- To detect and correct error(s) that is introduced during transmission of digital signal.



INTRODUCTION

- **Error control coding:**
Extra bits(one or more) are added to the data at the transmitter (redundancy) to permit error detection or correction at the receiver.
- **Classification of codes:**
 - 1) **Error detecting codes:** capable of only detecting the errors.
 - 2) **Error correcting codes:** capable of detecting as well as correcting the errors.



CLASSIFICATION OF ERROR CONTROL CODES

- Based upon memory:
 - Block code:** does not need memory.
 - Convolutional code:** needs memory.
- Based upon linearity:
 - Linear code**
 - Nonlinear code**



TYPES OF ERROR CONTROL

1. Automatic repeat request (ARQ) technique: receiver can request for the retransmission of the complete or a part of message if it finds some error in the received message. This requires an additional channel called feedback channel to send the receiver's request for retransmission.

Appropriate for

- Low delay channels
- Channels with a return path

Not appropriate for delay sensitive data, e.g., real time speech and data



TYPES OF ERROR CONTROL

2. **Forward error correction(FEC) technique:** no such feedback path and there is no request is made for retransmission.

- Coding designed so that errors can be corrected at the receiver
- Appropriate for delay sensitive and one-way transmission (e.g., broadcast TV) of data
- Two main types, namely block codes and convolutional codes



DRAWBACKS OF CODING TECHNIQUES

- Higher transmission bandwidth.
- System complexity.



IMPORTANT DEFINITIONS

- **Code word:** The code word is the n bit encoded block of bits. It contains message bits and parity or redundant bits.
- **Code rate/code efficiency:** It is defined as the ratio of the number of message bits(k) to the total number of bits(n) in a code word.

$$\text{Code rate (r)} = k/n$$

- **Hamming distance:** number of locations in which their respective elements differ.

e.g., 10011011

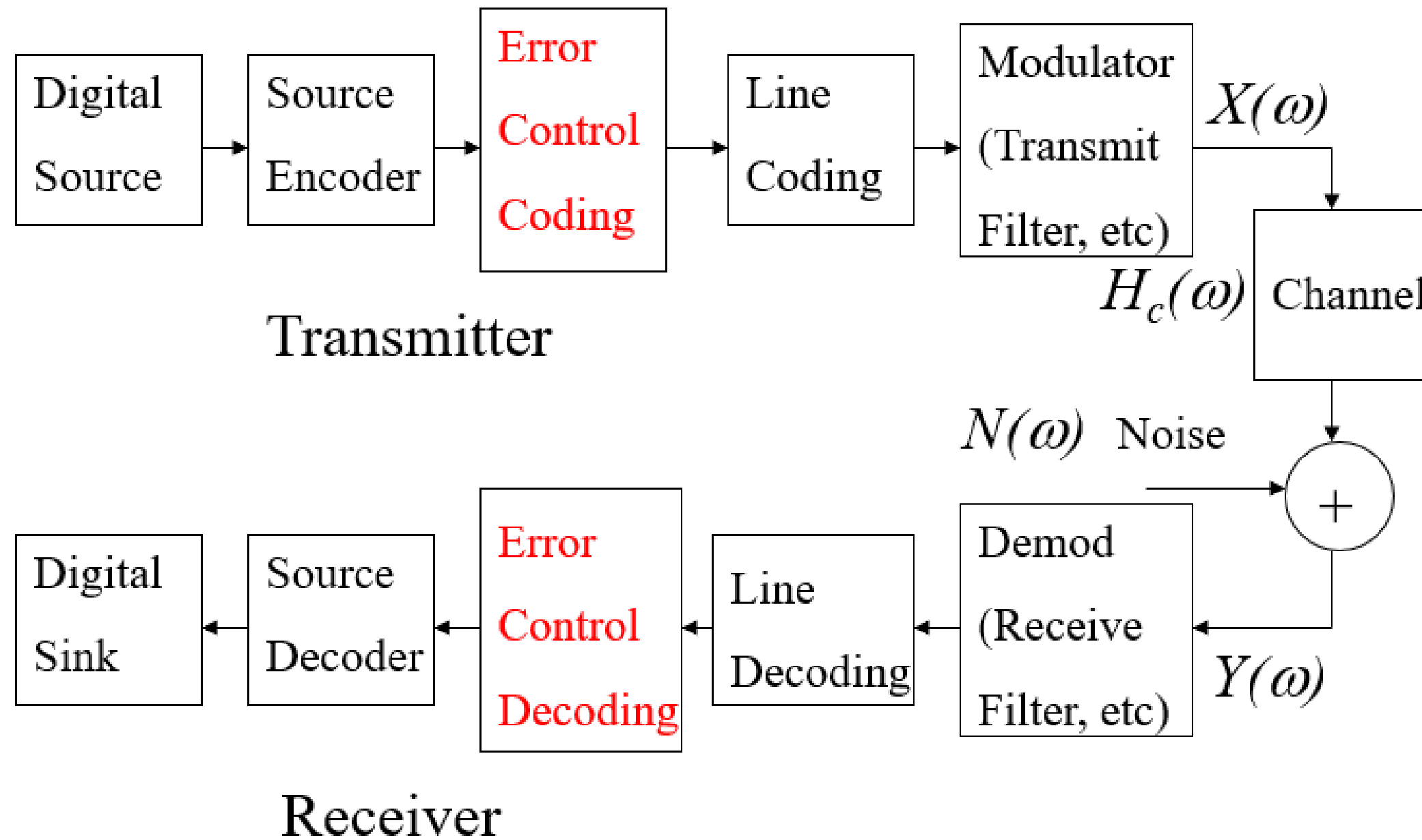
11010010 have a Hamming distance = 3

Alternatively, we can compute by adding code words (mod 2) =01001001 (now count up the ones)

- **Hamming weight of a code word:** It is defined as the number of nonzero elements in the code word.



TRANSMISSION MODEL





LINEAR BLOCK CODES

Definition: A code is said to be linear if any two code words in the code can be added in modulo 2 addition to produce a third code word in the code.

Code word length = n bits

$m_0, m_1, m_2, \dots, m_{k-1}$

$c_0, c_1, c_2, \dots, c_{n-k-1}$

k message bits

$(n-k)$ parity bits

(n, k) linear block code



LINEAR BLOCK CODES

- A vector notation is used for the message bits and parity bits
 - message bit $m = [m_0 m_1 \dots m_{k-1}]$
 - Parity bit $c = [c_0 c_1 \dots c_{n-k-1}]$



--The code vector can be mathematically represented by

$$X = [M:C]$$

M= k message vector

C= (n-k) parity vector



LINEAR BLOCK CODES

- A block code encoder generates the parity vector or parity bits required to be added to the message bits to generate the code word. The code vector x can also be represented as

$$[X]=[M][G]$$

X =code vector of $(1 \times n)$ size

M =message vector of $(1 \times k)$ size

G =generator matrix of $(k \times n)$ size

- The generator matrix depends on the type of linear block code used and is defined as

$$G = [I_k \mid P]$$

Where $I_k = (k \times k)$ identity matrix

$P = k \times (n - k)$ coefficient matrix



LINEAR BLOCK CODES



$$I_k = \begin{bmatrix} 1 & 0 & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{k \times k}$$



LINEAR BLOCK CODES



$$P = \begin{bmatrix} p_{00} & p_{10} & \dots & p_{n-k-1,0} \\ p_{01} & p_{11} & \dots & p_{n-k-1,1} \\ \cdot & \cdot & \dots & \cdot \\ p_{0,k-1} & p_{1,k-1} & \dots & p_{n-k-1,k-1} \end{bmatrix}_{k \times (n-k)}$$



LINEAR BLOCK CODES



- The parity vector can be obtained as

$$C=MP$$

$$\begin{bmatrix} c_0 & c_1 & \dots & c_{n-k-1} \end{bmatrix} = \begin{bmatrix} m_0 & m_1 & \dots & m_{k-1} \end{bmatrix} \begin{bmatrix} p_{00} & p_{10} & \dots & p_{n-k,0} \\ p_{01} & p_{11} & \dots & p_{n-k,1} \\ \cdot & \cdot & \dots & \cdot \\ p_{0,n-k} & p_{1,k-1} & \dots & p_{n-k,k-1} \end{bmatrix}$$



PARITY CHECK MATRIX (H)

- There is another way of expressing the relationship between the message bits and the parity bits of a linear block codes. Let H denote an $(n-k) \times n$ matrix defined as

$$H = [P^T \mid I_{n-k}]$$

Where $P^T = (n-k) \times k$ matrix representing the transpose of the coefficient matrix P

$I_{n-k} = (n-k) \times (n-k)$ identity matrix



ERROR DETECTION AND CORRECTION CAPABILITY OF LINEAR BLOCK CODE



- Hamming distance determines the error detecting and correcting capability of a linear block code.



ERROR DETECTION AND CORRECTION CAPABILITY OF LINEAR BLOCK CODE



- The maximum number of detectable errors is

$$d_{\min} - 1$$

- The maximum number of correctable errors is given by

$$t = \left\lfloor \frac{d_{\min} - 1}{2} \right\rfloor$$

where d_{\min} is the minimum Hamming distance between 2 code words and $\lfloor \cdot \rfloor$ means the largest integer less than or equal to the enclosed quantity.



PROPERTIES OF G AND H MATRIX



- $GH^T = 0$
- $HG^T = 0$
- $XH^T = 0$



Thank
you!