SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB301 – ANALOG AND DIGITAL COMMUNICATION

III B.E. ECE / V SEMESTER

UNIT 5 – INFORMATION THEORY AND ERROR CONTROL CODING

TOPIC – ERROR CONTROL CODING

Purpose

- To detect and correct error(s) that is introduced during transmission of digital signal.

INTRODUCTION

- **Error control coding:**

  Extra bits(one or more) are added to the data at the transmitter (redundancy) to permit error detection or correction at the receiver.

- **Classification of codes:**

  1) Error detecting codes: capable of only detecting the errors.

  2) Error correcting codes: capable of detecting as well as correcting the errors.

CLASSIFICATION OF ERROR CONTROL CODES

- Based upon memory:

  Block code: does not need memory.

  Convolutional code: needs memory.

- Based upon linearity:

  Linear code

  Nonlinear code

TYPES OF ERROR CONTROL

1. **Automatic repeat request(ARQ) technique**: receiver can request for the retransmission of the complete or a part of message if it finds some error in the received message. This requires an additional channel called feedback channel to send the receiver's request for retransmission.

Appropriate for

- Low delay channels
- Channels with a return path

Not appropriate for delay sensitive data, e.g., real time speech and data

TYPES OF ERROR CONTROL

2. Forward error correction(FEC) technique: no such feedback path and there is no request is made for retransmission.

- Coding designed so that errors can be corrected at the receiver
- Appropriate for delay sensitive and one-way transmission (e.g., broadcast TV) of data
- Two main types, namely block codes and convolutional codes

DRAWBACKS OF CODING TECHNIQUES

- Higher transmission bandwidth.
- System complexity.

IMPORTANT DEFINITIONS

- Code word: The code word is the n bit encoded block of bits. It contains message bits and parity or redundant bits.
- Code rate/code efficiency: It is defined as the ratio of the number of message bits(k) to the total number of bits(n) in a code word.

  Code rate (r) = k/n
- Hamming distance: number of locations in which their respective elements differ.

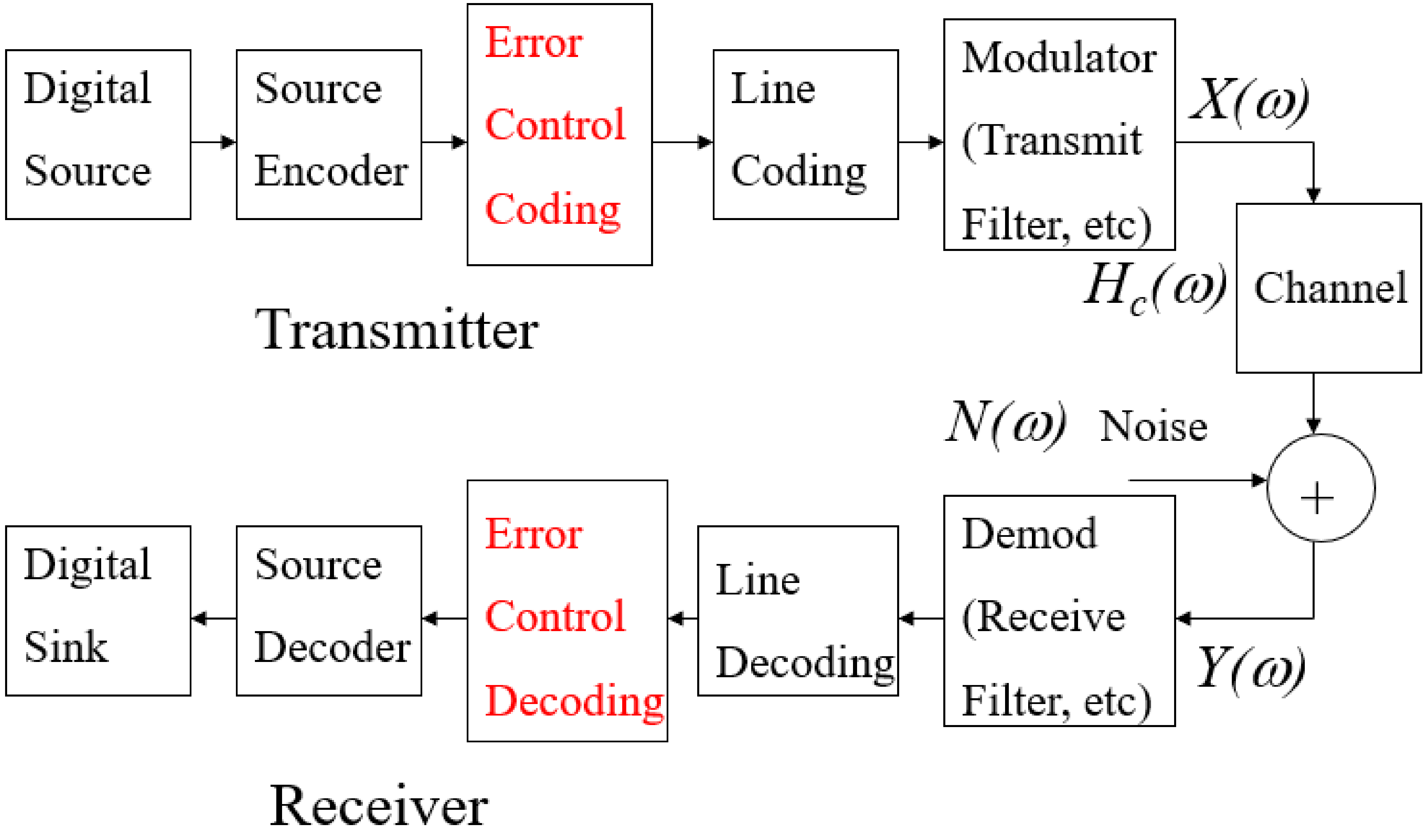
  e.g., 10011011

  11010010 have a Hamming distance = 3

  Alternatively, we can compute by adding code words (mod 2) =01001001 (now count up the ones)

- Hamming weight of a code word: It is defined as the number of nonzero elements in the code word.

TRANSMISSION MODEL



LINEAR BLOCK CODES

Definition: A code is said to be linear if any two code words in the code can be added in modulo 2 addition to produce a third code word in the code.

Code word length= n bits

$m_0, m_1, m_2 \ldots\ldots\ldots m_{k-1}$	$c_0, c_1, c_2 \ldots\ldots\ldots c_{n-k-1}$
k message bits	(n-k) parity bits

(n,k) linear block code

LINEAR BLOCK CODES

- A vector notation is used for the message bits and parity bits
  - message bit m = [$m_0\ m_1 \ldots . m_{k-1}$]
  - Parity bit c = [$c_0\ c_1 \ldots \ldots . c_{n-k-1}$]

--The code vector can be mathematically represented by

X=[M:C]

M= k message vector

C= (n-k) parity vector

LINEAR BLOCK CODES

- A block code encoder generates the parity vector or parity bits required to be added to the message bits to generate the code word. The code vector x can also be represented as

$$[X]=[M][G]$$

X=code vector of (1×n) size

M=message vector of (1×k) size

G=generator matrix of (k×n) size

- The generator matrix depends on the type of linear block code used and is defined as

$$G = [\ I_k \mid P]$$

Where I_k = (k×k) identity matrix

P= k×(n-k) coefficient matrix

LINEAR BLOCK CODES

$$I_k = \begin{bmatrix} 1 & 0 & . & . & 0 \\ 0 & 1 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{k \times k}$$

LINEAR BLOCK CODES

$$P = \begin{bmatrix} p_{00} & p_{10} & \cdots & p_{n-k-1,0} \\ p_{01} & p_{11} & \cdots & p_{n-k-1,1} \\ . & . & \cdots & . \\ p_{0,k-1} & p_{1,k-1} & \cdots & p_{n-k-1,k-1} \end{bmatrix}_{k\times(n-k)}$$

LINEAR BLOCK CODES

- The parity vector can be obtained as

C=MP

$$\begin{bmatrix} c_0 & c_1 \cdots\cdots c_{n-k-1} \end{bmatrix} = \begin{bmatrix} m_0 & m_1 \cdots\cdots m_{k-1} \end{bmatrix} \begin{bmatrix} p_{00} & p_{10} & \cdots & p_{n-k,0} \\ p_{01} & p_{11} & \cdots & p_{n-k,1} \\ . & . & \cdots & . \\ p_{0,n-k} & p_{1,k-1} & \cdots & p_{n-k,k-1} \end{bmatrix}$$

PARITY CHECK MATRIX (H)

- There is another way of expressing the relationship between the message bits and the parity bits of a linear block codes. Let H denote an (n-k)×n matrix defined as

$$H = [P^T \mid I_{n-k}]$$

Where P^T = (n-k)×k matrix representing the transpose of the coefficient matrix P

I_{n-k} = (n-k)×(n-k) identity matrix

ERROR DETECTION AND CORRECTION CAPABILITY OF LINEAR BLOCK CODE

- Hamming distance determines the error detecting and correcting capability of a linear block code.

ERROR DETECTION AND CORRECTION CAPABILITY OF LINEAR BLOCK CODE

- The maximum number of detectable errors is

$$d_{\min} - 1$$

- The maximum number of correctable errors is given by

$$t = \left\lfloor \frac{d_{\min} - 1}{2} \right\rfloor$$

where $d_{\min}$ is the minimum Hamming distance between 2 code words and $\lfloor \cdot \rfloor$ means the largest integer less than or equal to the enclosed quantity.

PROPERTIES OF G AND H MATRIX

- $GH^T = 0$
- $HG^T = 0$
- $XH^T = 0$

Thank you!