

Index p = no of +ve square terms in canonical form

$$= 2$$

Signature = Diff. b/w +ve square terms and -ve square terms.

$$= 2 - 0 = 2$$

Example: 4

Reduce the quadratic to canonical form of $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ and discuss about its nature.

Sol

The matrix form of the quadratic eqn is

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Step 1:

Find C.E., E.V. & E.V.

The characteristic equation is

$$\lambda^3 - D_1 \lambda^2 + D_2 \lambda - D_3 = 0$$

D_1 = Sum of the leading diagonal elements

$$= 1 + 2 + 1$$

D_2 = Sum of the minor of the leading "

$$= \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= (2-1) + (1) + (2-1)$$

$$= 3$$

$$D_3 = |A|$$

$$= \begin{vmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 1(2-1) + 1(-1+0)$$

$$= 1-1=0$$

$$\text{C.E. is } \lambda^3 - 4\lambda^2 + 3\lambda - 0 = 0$$

$$\lambda(\lambda^2 - 4\lambda + 3) = 0$$

$$\lambda = 0, \lambda = 1, \lambda = 3$$



The characteristic equation is

$$\lambda^3 - 4\lambda^2 + 3\lambda - 0 = 0$$

Eigen vectors

$$\begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Case 1) $\lambda = 0$

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cccc} 1 & -1 & 0 & | & 0 \\ -1 & 2 & 1 & | & 0 \\ 0 & 1 & 1 & | & 0 \end{array}$$

$$\frac{x_1}{-1+0} = \frac{x_2}{0-1} = \frac{x_3}{2-1} \Rightarrow \frac{x_1}{-1} = \frac{x_2}{-1} = \frac{x_3}{1}$$

$$x_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

When $\lambda = 3$

$$\begin{bmatrix} -2 & -1 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 - x_2 = 0$$

$$-x_1 - x_2 + x_3 = 0$$

$$\frac{x_1}{-1-0} = \frac{x_2}{0+2} = \frac{x_3}{2-1}$$

$$\frac{x_1}{-1} = \frac{x_2}{2} = \frac{x_3}{1}$$

$$x_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

When $\lambda = 1$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_2 = 0$$

$$-x_1 - x_2 + x_3 = 0$$

$$\frac{x_1}{-1-0} = \frac{x_2}{-0-0} = \frac{x_3}{0-1}$$

$$\frac{x_1}{-1} = \frac{x_2}{0} = \frac{x_3}{-1}$$

$$x_3 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

Step 6:

Canonical form

$$Y^T D Y, \text{ where } Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

\therefore The canonical form is $y_1^2 + y_2^2 + 3y_3^2$
 \therefore The nature of the quadratic form is
positive semi-definite.

$$\text{Rank}(A) = 2$$

$$\text{Index}(P) = 2$$

$$\text{Signature}(A) = 2$$

Example 5:

Reduce the quadratic to canonical form
 $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ and discuss about its
nature.