REDUCTION OF QUADRATIC EQUATION TO CANONICAL FORM

Defn:

A homogeneous polynomial of degree 2 with any number of variable are known as quadratic form

$$x^2+y^2+z^2+2xy+2yz+2zx=0$$

Formula for matrix form,

$$\begin{bmatrix} \text{coefficient of } x^2 & 1/2 \text{ coefficient of } xy & 1/2 \text{ coefficient } zx \\ 1/2 \text{ ,, } yx & \text{coefficient of } y^2 & \text{,, } yz \\ 1/2 \text{ ,, } zx & 1/2 \text{ ,, } zy & \text{coefficient of } z^2 \end{bmatrix}$$

canonical form:

If a quadratic form $Q = X^T A X$ can be reduced by a non-singular linear transformation $x = Ny$ to $Q = Y^T D Y$, where Y is column vector. then the form $Q = Y^T D Y$ is known as canonical form.

INDEX (P):

Index = No of +ve square terms in the canonical form.

Rank (r)

Rank = No. of non-zero eigenvalue in the canonical form

Signature (s)

Signature = Diff b/w no. of +ve & -ve square terms in the canonical form.

$$s = 2P - r$$

NATURE OF QUADRATIC FORM

i) Positive definite

All the eigen values of A are positive.

Example $\lambda : 1, 2, 3$

ii) Negative definite

All the eigen value of A are negative and

Example $\lambda = -1, -2, -3$

iii) Positive semi definite.

All the eigenvalue of A are non-negative and atleast one eigenvalue is zero.

Eg $\lambda = 0, 1, 2$

iv) Negative semi definite.

All the eigenvalues of A are non-positive and atleast one eigen value is zero

E.g $\lambda = 0, -1, -3$

v) Indefinite

Some eigenvalues are positive and some eigenvalues are negative

Eg $\lambda = -1, 1, 2$

$\lambda = -2, -1, 2$

REDUCE QUADRATIC FORM TO CANONICAL FORM

Example 1: Write the matrix of the quadratic form $2x_1^2 - 2x_2^2 + 4x_3^2 + 2x_1x_2 - 6x_1x_3 + 6x_2x_3$

Sol

$$Q = a_{11}x_1^2 + a_{22}x^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{23}x_1x_3 + 2a_{31}x_3x_1$$

$$\begin{bmatrix} 2 & 1 & -3 \\ 1 & -2 & 3 \\ -3 & 3 & 4 \end{bmatrix}$$