



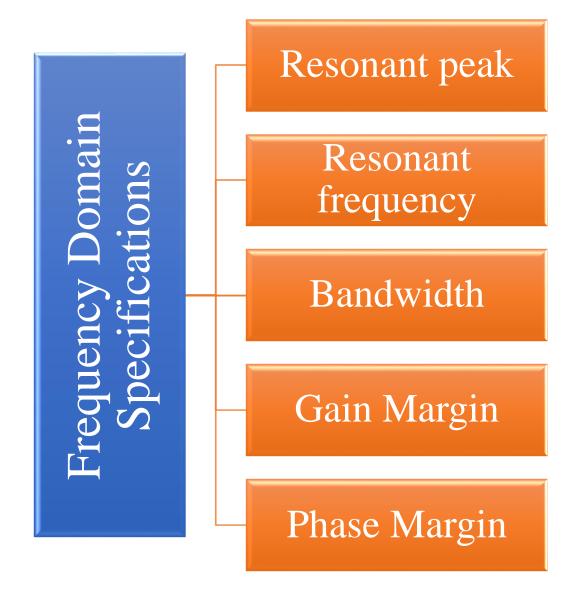


UNIT II FREQUENCY DOMAIN SPECIFICATIONS



INTRODUCTION









Gain Margin:

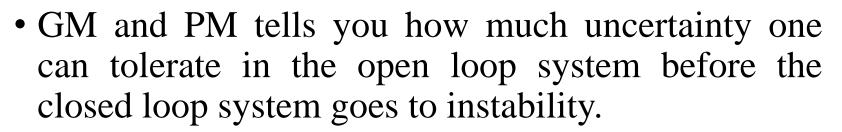
" It is the reciprocal of the magnitude of the open loop transfer function at phase cross over frequency. Phase cross over frequency is the frequency at which the phase angle is 180°".

Phase Margin:

" It is the additional phase lag at gain cross over frequency. Gain cross over frequency is the frequency at which the gain of $G(j\omega)$ is unity or 0db."







• Phase margin is related to closed loop damping ratio and so to the overshoot. To show this, consider an open loop system,

$$G_{oL}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$







$$G_{cL}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\begin{aligned} G(j\omega) &= \frac{\omega_n^2}{j\omega(j\omega+2\zeta\omega_n)} = \frac{-\omega_n^2}{\omega^2 - j2\zeta\omega_n\omega} \\ |G(j\omega)| &= \frac{\omega_n^2}{\sqrt{\omega^4 + 4\zeta^2\omega_n^2w^2}} \end{aligned}$$

Find ω_c , the crossover frequency.

$$\begin{array}{lcl} |G(j\omega_c)| &=& 1 \\ \Rightarrow \omega_n^4 &=& \omega_c^4 + 4\zeta^2 \omega_n^2 \omega_c^2 \\ \Rightarrow \omega_c &=& \omega_n \left[\sqrt{1 + 4\zeta^4} - 2\zeta^2 \right]^{1/2} \end{array}$$







Now, find the phase margin as follows,

$$PM = tan^{-1} \left[\frac{-\mathcal{I}m(G(j\omega))}{-\mathcal{R}e(G(j\omega))} \right]_{\omega = \omega_c}$$
$$= tan^{-1} \left[\frac{\mathcal{I}m(G(j\omega))}{\mathcal{R}e(G(j\omega))} \right]_{\omega = \omega_c}$$

Now,

$$G(j\omega) = \frac{-\omega_n^2}{\omega^2 - j2\zeta\omega_n\omega} = \frac{-\omega_n^2(\omega^2 + j2\zeta\omega_n\omega)}{\omega^4 - 2\zeta^2\omega_n^2\omega^2}$$

So,

$$PM = \tan^{-1} \left(\frac{2\zeta \omega_n \omega_c}{\omega_c^2} \right) = \tan^{-1} \left(\frac{2\zeta \omega_n}{\omega_c} \right)$$
$$= \tan^{-1} \left[\frac{2\zeta}{[\sqrt{1+4\zeta^4} - 2\zeta^2]^{1/2}} \right]$$