SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
COIMBATORE-35

UNIT III

FREQUENCY DOMAIN SPECIFICATIONS

INTRODUCTION

Frequency Domain Specifications

- Resonant peak
- Resonant frequency
- Bandwidth
- Gain Margin
- Phase Margin

GAIN AND PHASE MARGIN

Gain Margin:

" It is the reciprocal of the magnitude of the open loop transfer function at phase cross over frequency. Phase cross over frequency is the frequency at which the phase angle is 180°".

Phase Margin:

" It is the additional phase lag at gain cross over frequency. Gain cross over frequency is the frequency at which the gain of G(jω) is unity or 0db."

GAIN AND PHASE MARGIN

- GM and PM tells you how much uncertainty one can tolerate in the open loop system before the closed loop system goes to instability.
- Phase margin is related to closed loop damping ratio and so to the overshoot. To show this, consider an open loop system,

$$G_{oL}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

GAIN AND PHASE MARGIN

$$G_{cL}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega + 2\zeta\omega_n)} = \frac{-\omega_n^2}{\omega^2 - j2\zeta\omega_n\omega}$$

$$|G(j\omega)| = \frac{\omega_n^2}{\sqrt{\omega^4 + 4\zeta^2\omega_n^2 w^2}}$$

Find ω_c, the crossover frequency.

$$|G(j\omega_c)| = 1$$

$$\Rightarrow \omega_n^4 = \omega_c^4 + 4\zeta^2\omega_n^2\omega_c^2$$

$$\Rightarrow \omega_c = \omega_n\left[\sqrt{1 + 4\zeta^4} - 2\zeta^2\right]^{1/2}$$

GAIN AND PHASE MARGIN

Now, find the phase margin as follows,

$$\begin{aligned} \text{PM} &= tan^{-1}\left[\frac{-\mathcal{I}m(G(j\omega)}{-\mathcal{R}e(G(j\omega))}\right]_{\omega=\omega_c} \\ &= \tan^{-1}\left[\frac{\mathcal{I}m(G(j\omega))}{\mathcal{R}e(G(j\omega))}\right]_{\omega=\omega_c} \end{aligned}$$

Now,

$$G(j\omega) = \frac{-\omega_n^2}{\omega^2 - j2\zeta\omega_n\omega} = \frac{-\omega_n^2(\omega^2 + j2\zeta\omega_n\omega)}{\omega^4 - 2\zeta^2\omega_n^2\omega^2}$$

So,

$$\begin{aligned} \text{PM} &= \tan^{-1}\left(\frac{2\zeta\omega_n\omega_c}{\omega_c^2}\right) = \tan^{-1}\left(\frac{2\zeta\omega_n}{\omega_c}\right) \\ &= \tan^{-1}\left[\frac{2\zeta}{[\sqrt{1+4\zeta^4} - 2\zeta^2]^{1/2}}\right] \end{aligned}$$