Nature of the quadratic form

Find rank, index, signature and nature

$$2x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 = 0$$

The matrix form is

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Characteristic equation ,Eigen values,Eigen vectors

C_1 =Sum of leading diadonal elements

=2+2+1 =5

C_2= Sum of minors of leading diagonal elements

=4

$C_3=|A|$

$$= \begin{vmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= 0$

The characteristic equation is

$$\lambda^3 - 5\lambda^2 + 4\lambda = 0$$

The eigen values are 0,1,4

The index p=2

Rank r=2

Signature s=2p-r =2

The nature is semi positive