3.1 INTRODUCTION

Quantum Mechanics is the theoretical basis of Modern Physics that explains the nature and behavior of matter and energy on the atomic and subatomic level.

Classical Physics explains matter and energy at the macroscopic level of the scale familiar to human experience, including the behavior of astronomical bodies. At the end of the 19th century observers discovered phenomena in both the large and the small matters that Classical Physics could not explain.

Quantum Mechanics (also called wave mechanics) deals with the investigation of the behavior of micro-particles. Most advances that have taken place in Solid State Physics, Atomic Physics and Nuclear Physics are based on the principles of Quantum Mechanics.

There are five main ideas represented in quantum theory. They are

- Energy is not continuous.
- The elementary particles behave both like particles and like waves.
- The movement of these particles is inherently random.
- It is physically impossible to know both the position and the momentum of a particle at the same time with same accuracy.
- The atomic world is similar to our world which we live in.

We already know the basic concepts of particles and waves. A particle is described by its mass, velocity, momentum and energy. The spread out disturbance of the particle is named as wave and the wave can be described by its amplitude, frequency, wavelength, intensity and phase. In fact the electromagnetic radiation behaves as both particle and wave. Some of the phenomena like photoelectric effect, black body radiation, Compton's theory, Zeemen effect etc. are explained based on the particle (photon) nature of light. Hence radiation or light behaves like a wave as well as a particle. This character wave-particle of light radiation results the dual nature of light. However light radiations never possess both particle and wave characters simultaneously. Based on the above concept in 1924 Louis Victor due de Broglie, a French scientist proposed that particle like electrons, protons etc. should possess wave like properties when in motion. Based on the de Broglie's wave concept, in 1926, Schrodinger a German scientist derived two forms of wave equations, namely time-dependent and time-independent wave equation.

The Quantum Theory of black body radiation, Compton effect, de-Broglie waves, Schrödinger wave equations, various microscopes and their applications are explained in this unit.

3.2 BLACK BODY RADIATION

A perfect black body is one which absorbs all the thermal radiations incident upon it and it does not reflect light. There is no perfect black body exist. An object coated with a black pigment is nearly a black body.

Let us construct a classic experimental model as shown in Fig. 3.1. Here a hollow copper sphere is coated with lamp black on its inner wall. A fine hole is made for radiations to enter into the sphere.

Now the radiations are made to pass through the hole. The following phenomenon takes place.

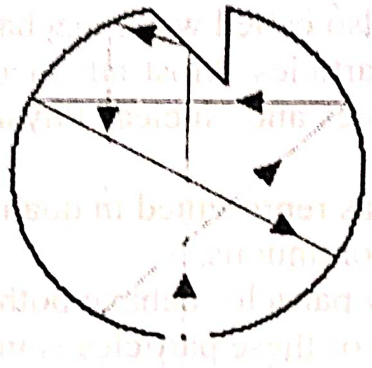

Fig. 3.1

- The radiation undergoes multiple reflections.
- The radiation is completely absorbed in the walls of the cavity.
- None of the radiation escapes.
- The radiation heats the cavity walls.
- Atoms in the walls of the cavity will vibrate.

When this black body is placed in a temperature bath of fixed temperature the atoms then re-radiate energy. The heat radiation will come out only through the hole in the sphere and not through the walls of the sphere. Hence we conclude that the radiations are emitted only from the inner surface of the sphere. The emission from a black body depends only on its temperature and it is independent of the material, shape and size of the body.

Therefore a perfect black body is a perfect absorber as well as a perfect radiator of all wavelengths.

3.2.1 Black Body Radiation Spectrum

The black body is allowed to emit radiations at different temperatures and the radiation curves are drawn (Fig.3.2). From these curves following results were observed.

- The black body emits radiation ranging from lower wavelength to higher wavelength.
- At a particular temperature the spectral radiation E_λ is a maximum at a particular wavelength called peak wavelength λ_m. This means that as

the temperature is raised the cavity emits more and more the radiation of the shorter wavelengths.

- The energy density E_λ increases with increase in wavelength and reacts a maximum and then decreases with increase in wavelength.
- For all the wavelengths an increase in temperature causes increase in energy.
- The total energy emitted at any particular temperature can be calculated from the area under that particular curve.

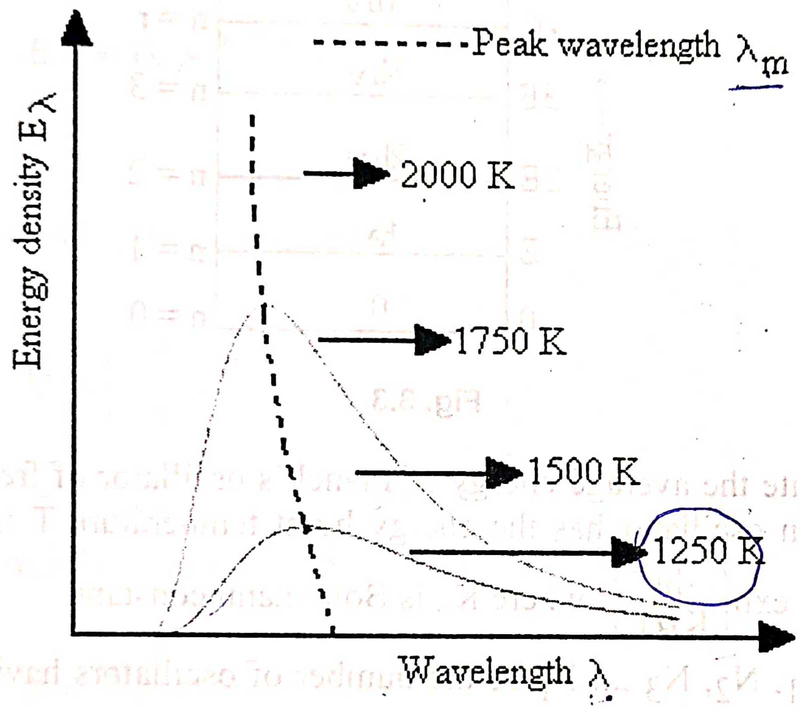

Fig. 3.2

3.3 PLANCK'S QUANTUM THEORY OF BLACK BODY RADIATION

In 1900, Planck introduced entirely new ideas to explain the distribution of energy among the various wavelengths of the cavity radiation.

Planck derived an expression for the energy distribution with the following assumptions.

1. He assumed that the atoms of the walls of the cavity radiator behave as oscillators, each with a characteristic frequency of oscillation.
2. The frequency of radiation emitted by an oscillator is the same as that of the frequency of its vibration of the radiator.
3. An oscillator can have only discrete energies given by

$$E = nh\nu$$

Where ν is the frequency of the oscillator, h is a constant known as Planck's constant and n is an integer known as quantum number. This means that the oscillator can have only energies hν, 2hν, 3hν and no energy in between.

4. The oscillators do not emit or absorb energy continuously but only in 'jumps' as shown in Fig. 3.3. That is an oscillator emits or absorbs packets of energy, each packet carrying an amount of energy hν.

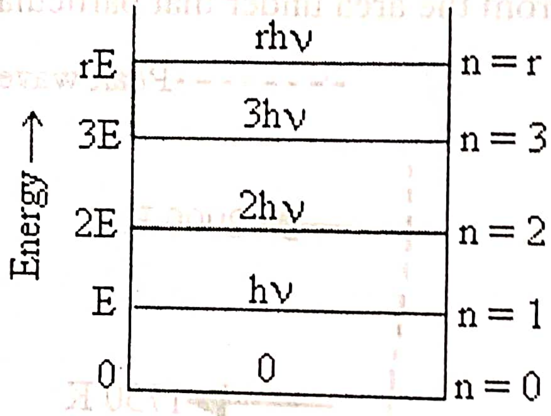

Fig. 3.3

Let us calculate the average energy of Planck's oscillator of frequency ν. The probability that an oscillator has the energy hν at temperature T is given by the Boltzmann factor $\exp\left(\frac{-h\nu}{K_B T}\right)$, where K_B is Boltzmann constant.

Let $N_0, N_1, N_2, N_3 \ldots N_r$ be the number of oscillators having energies 0, hν, 2 hν, 3 hν...r hν respectively.

Then we have $N_r = N_0 e^{-rE/K_BT}$

The total number of oscillators,

$$N = N_0 + N_1 + N_2 + N_3 + \ldots + N_r$$

$$N = N_0 \left[1 + e^{-E/K_BT} + e^{-2E/K_BT} + e^{-3E/K_BT} + \ldots + e^{-rE/K_BT}\right]$$

We know that $1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}$.

$$\therefore \quad N = N_0 \left[\frac{1}{1 - e^{-E/K_BT}}\right] \qquad \ldots\ldots(1)$$

Total energy of the oscillator is given by

$$E_T = (N_0\, 0) + (N_1\, h\nu) + (N_2\, 2h\nu) + (N_3 3\, h\nu) + \ldots..$$

$$E_T = (N_0\ 0) + (N_0 e^{\frac{-h\nu}{K_BT}}\ h\nu) + (N_0 e^{\frac{-2h\nu}{K_BT}}\ 2h\nu) + (N_0 e^{\frac{-3h\nu}{K_BT}}\ 3\ h\nu) + \ldots..$$

$$E_T = N_0 e^{\frac{-h\nu}{K_BT}} .h\nu\ (1 + 2e^{\frac{-h\nu}{K_BT}} + 3e^{\frac{-2h\nu}{K_BT}} + \ldots..)$$

We know $1 + 2x + 3x^2 + \ldots + r\, x^{r-1} = \frac{1}{(1-x)^2}$.

$$E_T = N_0\, e^{-h\nu/K_BT} .h\nu\left[\frac{1}{(1-e^{-h\nu/K_BT})^2}\right]$$

.....(2)

The average energy of an oscillator $\bar{E} = \frac{E_T}{N}$ is obtained by dividing eqn. (2) by eqn. (1)

$$\bar{E} = \frac{e^{-h\nu/K_BT}.h\nu}{1-e^{-h\nu/K_BT}}$$

$$\bar{E} = \frac{h\nu}{e^{h\nu/K_BT} - 1} \qquad .....(3)$$

The number of oscillators present/unit volume in the frequency range of ν and ν+ dν is given by

$$N = \frac{8\pi\nu^2}{c^3} d\nu \qquad .....(4)$$

The energy density of radiation $\rho_\nu d\nu$ in the frequency range ν to ν+dν

$$\rho_\nu d\nu = \begin{pmatrix}\text{Number of oscillators present/unit volume} \\ \text{in the frequency interval } d\nu\end{pmatrix} X \begin{pmatrix}\text{Average energy} \\ \text{of the oscillator}\end{pmatrix}$$

$$\rho_\nu d\nu = \frac{8\pi\nu^2}{c^3} d\nu \times \bar{E}$$

$$\therefore \quad \rho_\nu d\nu = \frac{8\pi h\nu^3}{c^3} . \frac{1}{e^{h\nu/K_BT} - 1} d\nu \qquad .....(5)$$

Equation (5) is known as Planck's equation for black body radiation.

Planck's radiation law in terms of wavelength (λ)

The frequency is given by $\nu = \frac{c}{\lambda}$ .....(6)

Differentiating we get $d\nu = \frac{-c}{\lambda^2} d\lambda$

$$\therefore \quad |d\nu| = \left[\frac{c}{\lambda^2}\right] d\lambda \qquad .....(7)$$

Substituting the values of ν and $d\nu$ in RHS of equation (5) we get,

The energy density $\rho_\lambda d\lambda = \frac{8\pi h c^3}{c^3 \lambda^3}\left(\frac{c}{\lambda^2}\right) d\lambda \left(\frac{1}{e^{hc/\lambda K_B T} - 1}\right)$

$$\therefore \quad \rho_\lambda = \frac{8\pi h c}{\lambda^5}\left(\frac{1}{e^{hc/\lambda K_B T} - 1}\right) \qquad .....(8)$$

Equation (8) represents the Planck's radiation law in terms of wavelength. This law has good agreement with all the experimental results. It also helps to derive the Stefan – Boltzmann's law, Wein's displacement law and Rayleigh – Jean's law.

3.3.1 Wein's Displacement Law

Wein's displacement law holds good only for shorter wavelength region of the black body radiation.

The Planck's law is

$$\rho_\lambda = \frac{8\pi h c}{\lambda^5}\left(\frac{1}{e^{hc/\lambda K_B T} - 1}\right)$$

If λ is less $\frac{1}{\lambda}$ will be greater. $\therefore\ e^{hc/\lambda K_B T} >> 1$

Then $e^{hc/\lambda K_B T} - 1 \approx e^{hc/\lambda K_B T}$

Planck's law becomes $\quad \rho_\lambda = \frac{8\pi hc}{\lambda^5 (e^{hc/\lambda K_B T})}$

(or)

$$\rho_\lambda = 8\pi hc\, \lambda^{-5} e^{-hc/\lambda K_B T}$$

$$\rho_\lambda = C_1 \lambda^{-5} e^{(-C_2/\lambda T)} \qquad .....(9)$$

Where C_1 and C_2 are constants given by $C_1 = 8\pi hc$ and $C_2 = \frac{hc}{K_B}$

Equation (9) is the Wein's displacement law.

3.3.2 Rayleigh-Jean's law

We know Rayleigh-Jeans law holds good only for longer wavelength region of the black body radiation.

The Planck's law is

$$\rho_\lambda = \frac{8\pi h c}{\lambda^5}\left(\frac{1}{e^{hc/\lambda K_B T} - 1}\right)$$

If 'x' is small then $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

Here 'x' is small hence neglecting the higher order terms $e^x = 1 + x$

$$\therefore \quad e^{hc/\lambda K_B T} = 1 + \frac{hc}{\lambda K_B T}$$

The Planck's law becomes

$$\rho_\lambda = \frac{8\pi h c}{\lambda^5}\left(\frac{1}{1 + \frac{hc}{\lambda K_B T} - 1}\right)$$

$$\rho_\lambda = \frac{8\pi h c \lambda K_B T}{hc\lambda^5}$$

$$\rho_\lambda = \frac{8\pi K_B T}{\lambda^4} \qquad \ldots\ldots(10)$$

Equation (10) is the Rayleigh-Jean's law.

3.4 CHARACTERISTICS OF PHOTON

(i) Max Planck introduced the concepts that emission or absorption of electromagnetic radiation takes place as discrete quanta or tiny discrete packets called 'photons' each having an energy hν where h is the Planck's constant and ν is the frequency of radiation.

(ii) Photons are not affected by electric and magnetic field and they are electrically neutral.

(iii) Mass of the photon $m = h/c\lambda$ and momentum of the photon $P = h/\lambda$ since $P = mc$.

(iv) Velocity of the photon is 3×10^8 m/sec and it is equal to the velocity of light.

3.5 COMPTON EFFECT AND COMPTON SHIFT

In 1921 A. H. Compton explained the behavior of the monochromatic radiation scattered by a substance on the basis of quantum theory of radiation.

When a monochromatic beam of x-rays is scattered by a substance, the scattered x-rays contain radiation not only of the same wavelength as that of unmodified primary radiation but also the modified radiation of longer wavelength. This is called 'Compton effect'. The difference between scattered wavelengths is called 'Compton shift'.

When a photon of energy 'hv' collides with an electron of a scatterer at rest, the photon gives its energy to the electron. Therefore the scattered photon will have lesser energy (or) lower frequency (or) higher wavelength compared to the wavelength of incident photon. Since the electron gains energy, it recoils with the velocity 'v'. This effect is called Compton effect and the shift in wavelength is called **Compton shift**. Thus as a result of Compton scattering we get (i) unmodified radiations (ii) modified radiations and (iii) a recoil electron.

The Classical electromagnetic theory failed to explain the presence of the modified radiation. Compton satisfactorily explained the modified radiations on the basis of quantum theory.

According to Compton the primary x-ray beam is made up of photons of energy hv, where h is the Planck's constant and v is the frequency of primary x-ray radiation. The photons travel with velocity of light and possess momentum given by hv/c. According to the Classical theory radiations exerts pressure and possess momentum P= E/c where E is the energy of the radiation and c is the velocity of light. Now for a photon energy E= hv so that the momentum of the photon P= hv/c.

3.6 THEORY OF COMPTON EFFECT

Principle

In Compton scattering the collision between a photon and an electron is considered. Then by applying the laws of conservation of energy and momentum, the expression for Compton wavelength is derived.

Assumptions

i. The collision occurs between the photon and an electron in the scattering material.

ii. The electron is free and is at rest before collision with the incident photon.

With these two assumptions let us consider a photon of energy 'hv' and momentum P = hv/c colliding with an electron at rest which is in the origin 'O'. During the collision process a part of photon energy is given to the electron which in turn increases the kinetic energy of the electron and hence it recoils at an angle of ϕ as shown in Fig. 3.4. The scattered photon moves with an energy hv' and momentum P = hv'/c at an angle θ with respect to the original direction.

Let us find the energy and momentum components before and after collision process.

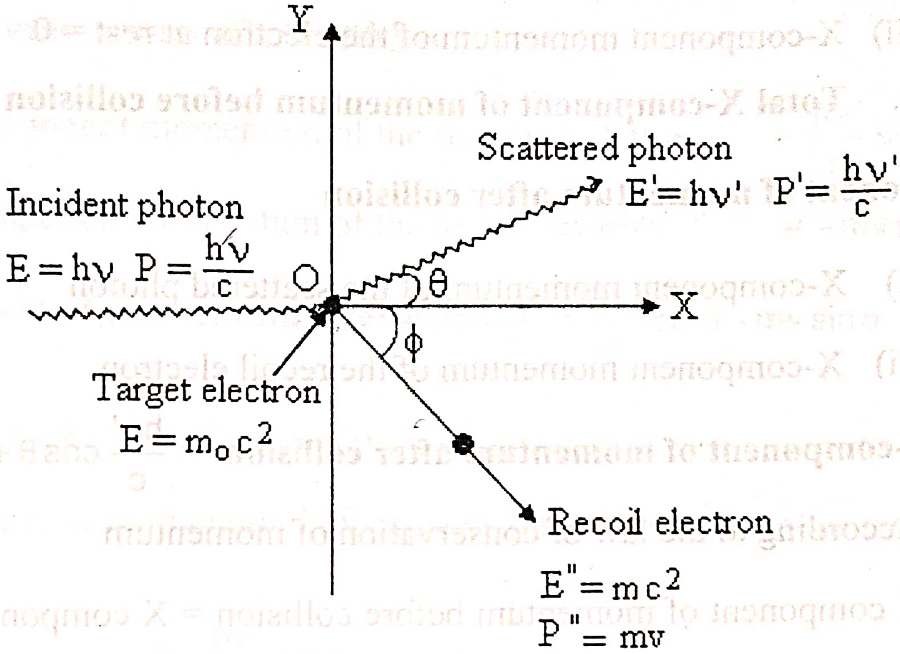

Fig. 3.4

Energy before collision

(i) Energy of the incident photon $= h\nu$

(ii) Energy of the electron at rest $= m_0c^2$

Where 'm_o' is the rest mass of the electron.

$\therefore$ **Total energy before Collision** $= h\nu + m_0c^2$ .....(1)

Energy after collision

(i) Energy of the scattered photon $= h\nu'$

(ii) Energy of the recoil electron $= mc^2$

Where 'm' is the mass of the electron which is moving with velocity 'c'

$\therefore$ **Total energy after Collision** $= h\nu' + mc^2$ .....(2)

According to the law of conservation of energy,

Total energy before collision = Total energy after collision

$$\therefore \quad h\nu + m_0c^2 = h\nu' + mc^2 \quad .....(3)$$

Calculations of momentum (both X-component and Y-component) before and after collisions are represented pictorially as shown in Fig. 3.5.

X-component of momentum before collision

(i) X-component momentum of the incident photon $= \frac{h\nu}{c}$

(ii) X-component momentum of the electron at rest = 0

$\therefore$ **Total X-component of momentum before collision =** $\frac{h\nu}{c}$ ....(4)

X-component of momentum after collision

(i) X-component momentum of the scattered photon $= \frac{h\nu'}{c}\cos\theta$

(ii) X-component momentum of the recoil electron $= mv\cos\phi$

Total X-component of momentum after collision $= \frac{h\nu'}{c}\cos\theta + mv\cos\phi$ ...(5)

According to the law of conservation of momentum

X component of momentum before collision = X component of momentum after collision

$$\therefore \quad \frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \qquad .....(6)$$

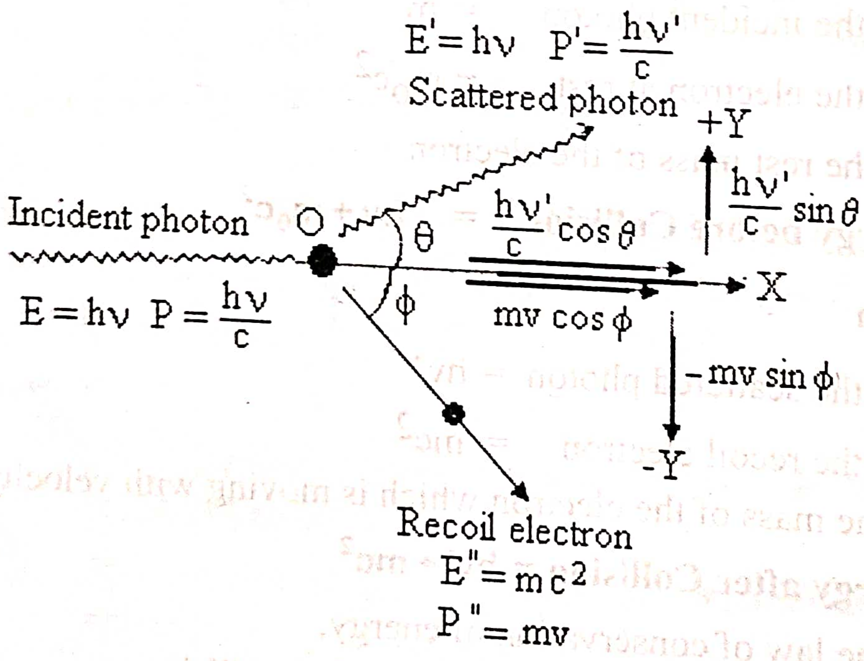

Fig. 3.5

Y-Component of momentum before collision

(i) Y-component momentum of the incident photon = 0

(ii) Y-component momentum of the electron at rest = 0

Total Y-component of momentum before collision = 0 .....(7)

Y-component of momentum after collision

(i) Y-component momentum of the scattered photon $= \frac{h\nu'}{c}\sin\theta$

(ii) Y-component momentum of the recoil electron $= -mv\sin\phi$

Total Y-component of momentum after collision $= \frac{h\nu'}{c}\sin\theta - mv\sin\phi$ .....(8)

According to the law of conservation of momentum

Y component of momentum before collision = Y component of momentum after collision

$$\therefore 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi \quad .....(9)$$

Equation (6) can be written as

$$\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta = mv\cos\phi$$

(or) $$mcv\cos\phi = h(\nu - \nu'\cos\theta) \quad .....(10)$$

Equation (9) can be written as

$$mcv\sin\phi = h\nu'\sin\theta \quad .....(11)$$

Squaring and adding equations (10) and (11) we get

$$m^2c^2v^2(\cos^2\phi + \sin^2\phi) = h^2[\nu^2 - 2\nu\nu'\cos\theta + (\nu')^2\cos^2\theta] + h^2(\nu')^2\sin^2\theta$$

Here $\cos^2\phi + \sin^2\phi = 1$ and $h^2(\nu')^2[\cos^2\theta + \sin^2\theta] = h^2(\nu')^2$ then

$$m^2c^2v^2 = h^2[\nu^2 - 2\nu\nu'\cos\theta + (\nu')^2] \quad .....(12)$$

Equation (3) can be written as

$$mc^2 = m_0c^2 + h[\nu - \nu']$$

Squaring on both sides we get

$$m^2c^4 = m_0^2c^4 + 2hm_0c^2(\nu - \nu') + h^2[\nu^2 - 2\nu\nu' + (\nu')^2] \quad .....(13)$$

Subtracting equation (12) from equation (13) we obtain

$$m^2c^2(c^2 - v^2) = m_0^2c^4 + 2hm_0c^2(\nu - \nu') - 2h^2\nu\nu'(1 - \cos\theta) \quad ....(14)$$

From the theory of relativity the relativistic formula for the variation of mass with the velocity of the electron is given by,

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Squaring on both sides we get $m^2 = \frac{m_0^2}{\left(1-\frac{v^2}{c^2}\right)}$ (or) $m^2 = \frac{m_0^2 c^2}{c^2 - v^2}$

or $$m^2 (c^2 - v^2) = m_0^2 c^2 \qquad .....(15)$$

In order to make this equation similar to LHS of equation (14) multiply it by c^2 on both sides.

$$m^2c^2(c^2 - v^2) = m_0^2 c^4 \qquad .....(16)$$

Equating equations (16) and (14) we get

$$m_0^2 c^4 = m_0^2 c^4 + 2hm_0c^2(\nu - \nu') - 2h^2\nu\nu'(1-\cos\theta)$$

or $$2hm_0c^2(\nu - \nu') = 2h^2\nu\nu'(1-\cos\theta)$$

or $$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

or $$\frac{\nu}{\nu\nu'} - \frac{\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

or $$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)$$

Multiplying both sides by 'c' then

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{hc}{m_0c^2}(1-\cos\theta) \qquad .....(17)$$

Since $\lambda = \frac{c}{\nu}$ and $\lambda' = \frac{c}{\nu'}$, we can write equation (17) as,

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$$

Here λ and λ' are the wavelengths of the incident and scattered photons.

Change in wavelength

$$\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta) \qquad .....(18)$$

This is the expression for the change in wavelength. $\Delta\lambda$ is called Compton shift. The change in wavelength or Compton shift depends neither or the incident wavelength nor or the scattering material, but depends only on the angle of incidence.

Equation (18) represents the shift in wavelength, i.e., Compton shift which is independent of the incident radiation as well as the nature of the scattering substance. Thus, the shift in wavelength or Compton shift purely depends on the angle of scattering.

Case (i) When $\theta = 0$; $\Delta\lambda = 0$ there is no scattering of photon along the incident radiation.

Case (ii) When $\theta = \frac{\pi}{2}$; $\Delta\lambda = \frac{h}{m_0 c}$

$$\Delta\lambda = \frac{6.625 \times 10^{-34}}{(9.11 \times 10^{-31})(3 \times 10^{8})}$$

$$\Delta\lambda = 0.02424 Å$$

This is called Compton wavelength.

Case (iii) When $\theta = \pi$; $\Delta\lambda = \frac{2h}{m_0 c}$

$$\Delta\lambda = 0.04848 Å$$

$\Delta\lambda$ is maximum value at $\theta = \pi$ and is equal to the twice the Compton wavelength.

3.7 EXPERIMENTAL VERIFICATION OF COMPTON EFFECT

Compton experiment established the validity of Compton's theory as well as the quantum theory of radiation. It also provides a good check of the particle concepts of photon.

X-rays of monochromatic wavelength 'λ' is produced from an X-ray Coolidge tube and is made to pass through the slits S_1 and S_2 as shown in Fig. 3.6. These X-rays are made to fall on the scattering element. The scattered X-rays are received with the help of the Bragg's spectrometer and the scattered wavelength is measured.

The experiment is repeated for various scattering angles and the scattered wavelengths are measured. The experimental results are plotted as shown in Fig. 3.7.

In Fig. 3.7 when the scattering angle $\theta = 0^{\circ}$ the scattered radiation peak will be the same as that of the incident radiation Peak-A. Now when the scattering angle is increased for one incident radiation of wavelength (λ) we get two scattered Peaks-A and B. Here the Peak-A is found to be of same wavelength as that of the incident wavelength and the Peak-B is of greater wavelength than the incident radiation.

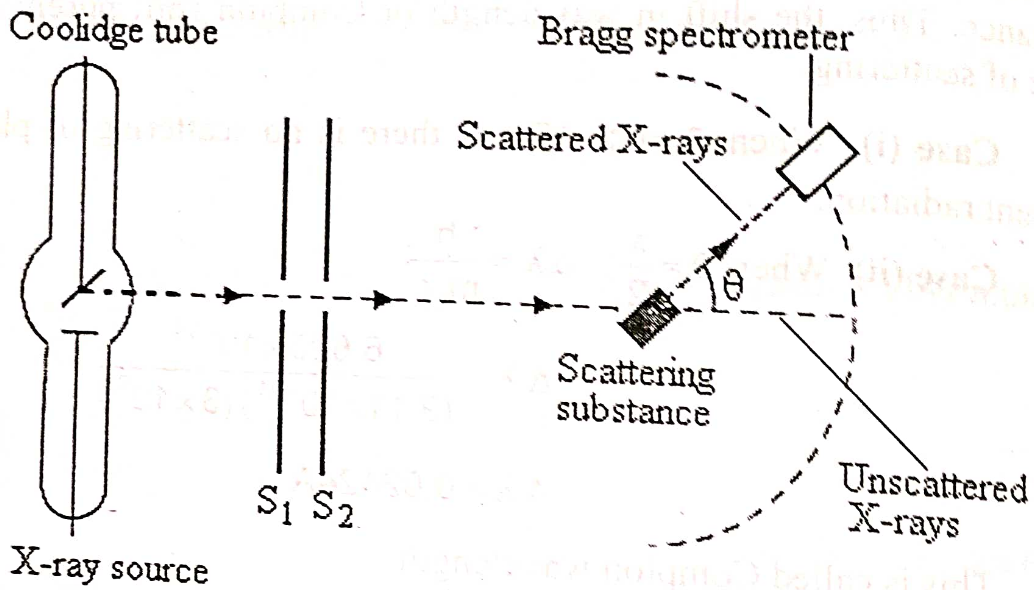

Fig. 3.6

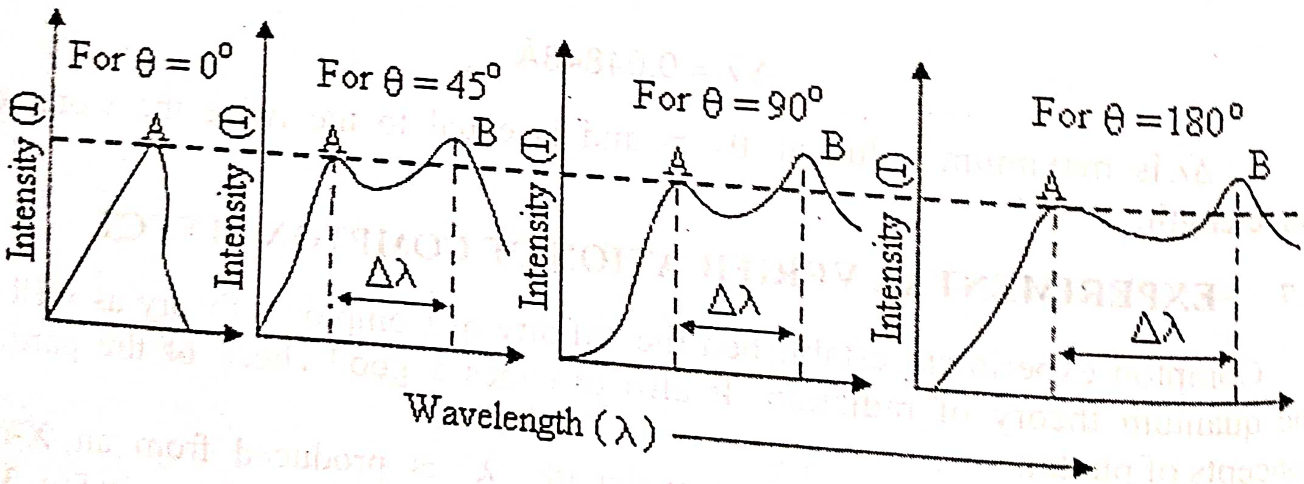

Fig. 3.7

The shift in wavelength or difference in wavelength $(\Delta\lambda)$ of the two scattered beams is found to increase with respect to the increase in scattering angle.

At $\theta = 90°$ the wavelength of the unmodified line is 0.708 Å whereas the wavelength of the modified line is 0.730Å. The change in wavelength 0.022Å which has good agreement with the theoretical value 0.024Å. This wavelength is called **Compton wavelength** and the shift in wavelength is called **Compton shift.**

3.8 DUAL NATURE OF RADIATION AND MATTER

In 1924, Louis Victor due de Broglie a French scientist proposed that light waves behaves sometimes as particles and some other times behaves as waves. Hence the material particles like atoms, molecules, electrons, protons or neutrons behave as waves are known as matter waves or de Broglie waves. This concept of particle-wave duality is called De-Broglie hypothesis.

Matter waves or de-Broglie waves

The waves associated with moving material particles are called De-Broglie waves.

3.8.1 de-Broglie wavelength

According to Einstein's mass-energy relation the energy of the particle is given by,

$$E = mc^2 \qquad \text{......(1)}$$

Where m is the mass of the particle and c is the velocity of light.

Equation (1) corresponds to the **particle nature.**

According to Planck's theory of radiation the energy of the photon is given by

$$E = h\nu \qquad \text{.....(2)}$$

Where h is the Planck's constant and ν is the frequency of radiation which is equal to c/λ.

Equation (2) corresponds to the **wave nature.**

From equations (1) and (2) we get

$$h\nu = mc^2 \qquad \text{.....(3)}$$

$$h\frac{c}{\lambda} = mc^2$$

$$\therefore \quad \lambda = \frac{h}{mc}$$

$$\lambda = \frac{h}{p} \qquad \text{.....(4)}$$

Eqn. (4) represents the de-Broglie wavelength for a photon of momentum p.

de-Broglie suggested that equation (4) can also be applied for material particles.

If 'm' is the mass of the particle and 'v' is the velocity of the particle then the momentum of the particle p = mv. In general the de Broglie wavelength

$$\lambda = \frac{h}{mv} \quad .....(5)$$

3.8.2 de-Broglie Wavelength of an electron

If an electron of mass 'm' and charge 'e' is accelerated by a voltage V then the work done on the electron to obtain the velocity 'v' is eV. This energy must be equal to the kinetic energy $\frac{1}{2}mv^2$.

$$\therefore \quad eV = \frac{1}{2}mv^2$$

Multiplying by 'm' on both sides we get

$$2\,meV = m^2v^2$$

or

$$mv = \sqrt{2meV} \quad .....(6)$$

Substituting equation (6) in (5) we get

de-Broglie wavelength of an electron

$$\lambda_e = \frac{h}{\sqrt{2meV}} \quad .....(7)$$

We know that the rest mass of the electron m = 9.11×10^{-31} kg, Planck's constant h = 6.62×10^{-34} J/sec and the electronic charge e = 1.602×10^{-19} C then the above equation (7) can be written as

$$\lambda_e = \frac{12.27}{\sqrt{V}}\text{Å} \quad .....(8)$$

3.8.3 de-Broglie wavelength in terms of kinetic energy

We know that the kinetic energy

$$E = \frac{1}{2}mv^2$$

Multiplying by 'm' on both sides we get

$$Em = \frac{1}{2}m^2v^2$$

or

$$m^2v^2 = 2\,Em$$

$$mv = \sqrt{2\,Em} \quad .....(9)$$

Substituting eqn.(9) in (5) we get

de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}} \qquad .....(10)$$

3.9 PROPERTIES OF MATTER WAVES

1. Greater the mass of the particle smaller will be the wavelength of the matter waves.
2. Matter waves are not electromagnetic waves.
3. Matter waves can travel faster than light.

3.10 G. P. THOMSON EXPERIMENT

When material particle have a wave nature then they are expected to show the interference and diffraction effects. In 1928 G.P. Thomson demonstrated that a beam of electrons does suffer diffraction. G.P. Thomson's apparatus for the diffraction of electron is shown Fig. 3.8.

The whole Thomson's apparatus is highly evacuated. The electrons are emitted by the cathode C and accelerated through a high positive potential given to the anode. Then the accelerated electron beam is passed through the slits and falls on a gold foil F of thickness 10^{-8}m. The electrons which are passing through the gold foil are received on a photographic plate.

In order to check whether the diffraction pattern is produced by the electron or by the x-rays a magnetic field is applied between the gold foil and the photographic plate. It is noted that there is deflection in the path of the electron indicating that the diffraction pattern is produced by electrons not by x-rays because x-rays are not affected by electric and magnetic field.

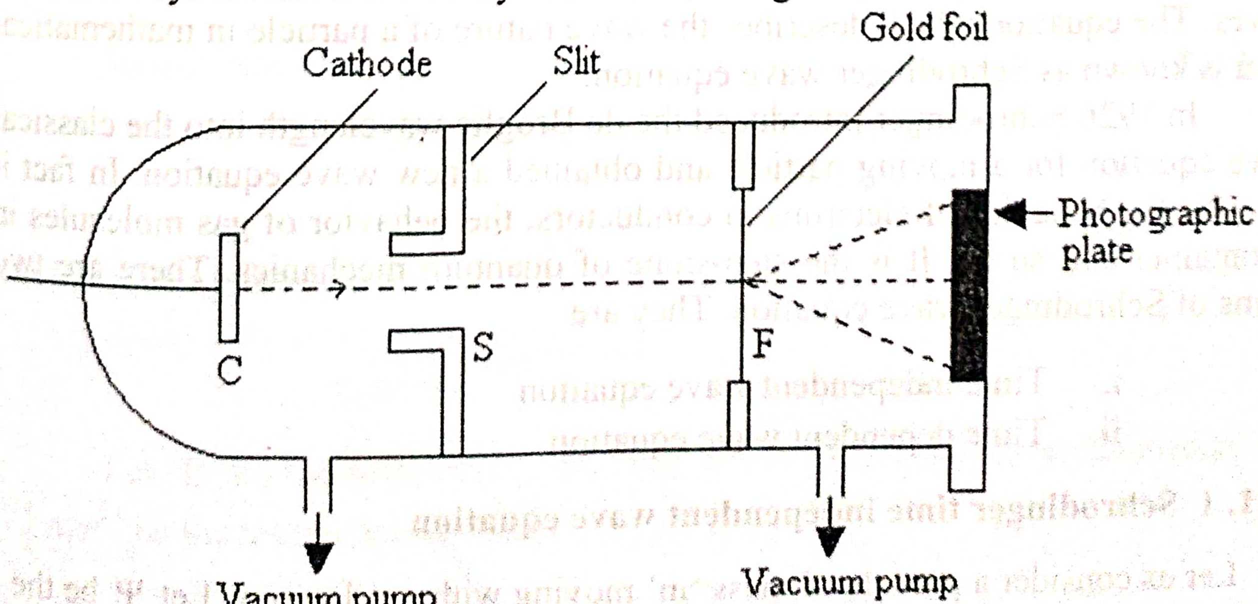

Fig. 3.8

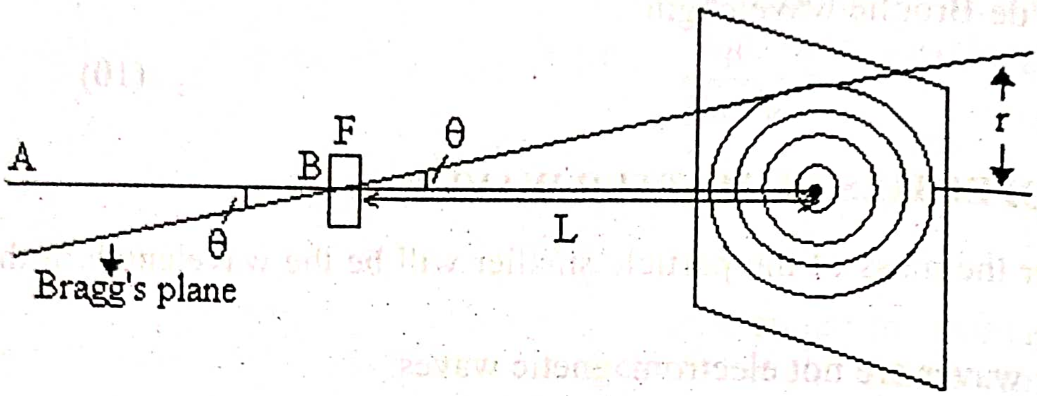

Fig. 3.9

After developing the photographic plate a symmetrical pattern consisting of concentric rings about a central spot is obtained as shown in Fig. 3.9. This pattern is similar to that produced by X- rays.

Hence Thomson confirms that the circular diffraction pattern is due to the wave nature of electrons. Thomson calculated the wavelength of the electron from de Broglie equation $\lambda_e = \frac{12.27}{\sqrt{V}}$ Å and the interplanar spacing from Bragg's equation $\lambda = 2d \sin\theta$ for $n = 1$ where d is the crystal lattice spacing.

Hence Thomson experiment clearly demonstrated the existence of de Broglie's matter waves.

3.11 SCHRODINGER WAVE EQUATION

Schrodinger equation is one of the basic equations in quantum mechanics. This equation can be applied for both macroscopic and microscopic particles. Schrodinger derived a mathematical equation to describe the dual nature of matter waves. The equation which describes the wave nature of a particle in mathematical form is known as Schrodinger wave equation.

In 1926 Schrodinger introduced the de Broglie wavelength into the classical wave equation for a moving particle and obtained a new wave equation. In fact it describe the behavior of electrons in conductors, the behavior of gas molecules in a container and so on. It is the step-stone of quantum mechanics. There are two forms of Schrodinger wave equation. They are

i. Time independent wave equation
ii. Time dependent wave equation

3.11. 1 Schrodinger time independent wave equation

Let us consider a particle of mass 'm' moving with a velocity v. Let Ψ be the wave function of the particle along x, y and z coordinate axes at any instant. The de-Broglie wavelength associated with it is given by $\lambda = \frac{h}{mv}$.

We know that the classical differential equation of a progressive wave moving with a wave velocity 'v' is

$$\frac{\partial^2 \Psi}{\partial x^2}+\frac{\partial^2 \Psi}{\partial y^2}+\frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} \quad .....(1)$$

The general solution for equation (1) is

$$\Psi = \Psi_o e^{-i\omega t} \quad .....(2)$$

Where Ψ_o is the amplitude of the wave at a point (x,y,z). It is a function of position. Differentiating equation (2) with respect to 't' twice we get

$$\frac{\partial^2\Psi}{\partial t^2} = -\omega^2\Psi \quad .....(3)$$

Substituting equation (3) in (1) we get

$$\frac{\partial^2 \Psi}{\partial x^2}+\frac{\partial^2 \Psi}{\partial y^2}+\frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{v^2}(-\omega^2\Psi)$$

Introducing the Laplacian operator $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ then

$$\nabla^2 \Psi = -\frac{\omega^2}{v^2}\Psi \quad .....(4)$$

We know that $\omega = 2\pi\nu = 2\pi(v/\lambda)$

or $$\frac{\omega}{v} = \frac{2\pi}{\lambda} \quad .....(5)$$

Substituting equation (5) in (4) we get

$$\nabla^2\Psi + \frac{4\pi^2}{\lambda^2}\Psi = 0 \quad .....(6)$$

On introducing the de Broglie's wave concept $\lambda = \frac{h}{mv}$ in equation (6) we get

$$\nabla^2\Psi + \frac{4\pi^2 m^2 v^2}{h^2}\Psi = 0 \quad .....(7)$$

Let 'E' be the total energy of the particle, 'V' be the potential energy and '$\frac{1}{2}mv^2$' be the kinetic energy then

$$\text{Total Energy } E = \text{P.E.} + \text{K.E.}$$

$$E = V + \tfrac{1}{2}mv^2$$

$$E - V = \frac{1}{2} mv^2$$

$$2(E - V) = mv^2$$

Multiplying by 'm' on both sides in the above equation we get,

$$2m (E - V) = m^2 v^2 \qquad .....(8)$$

Substituting equation (8) in (7) we obtain

$$\nabla^2\Psi + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0 \qquad .....(9)$$

This equation is known as Schrodinger time independent three dimensional wave equation.

From the above equation the one dimensional Schrodinger time independent wave equation can be written as

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0 \qquad .....(10)$$

Introducing $\hbar = \frac{h}{2\pi}$ in equation (9) we obtain another form of Schrodinger time independent three dimensional wave equation

$$\nabla^2\Psi + \frac{2m}{\hbar^2}(E - V)\Psi = 0 \qquad .....(11)$$

Special case:

For a free particle the potential energy V = 0 then the Schroedinger wave equation for a free particle is

$$\nabla^2\Psi + \frac{2mE}{\hbar^2}\Psi = 0 \qquad .....(12)$$

3.11. 2 Schrodinger time dependent wave equation

By eliminating E in the Schrodinger time independent wave equation we get the Schrodinger time dependent wave equation.

Differentiating equation (2) with respect to 't' we get

$$\frac{\partial\Psi}{\partial t} = -i\omega\Psi_o e^{-i\omega t}$$

$$= -i(2\pi\nu)\Psi_o e^{-i\omega t} \qquad \because \omega = 2\pi\nu$$

$$= -i2\pi\nu\Psi \qquad \because \Psi = \Psi_o e^{-i\omega t}$$

$$= -i2\pi\frac{E}{h}\Psi \qquad \because E = h\nu \text{ or } \nu = \frac{E}{h}$$

$$= -i\frac{E}{\hbar}\Psi$$

or $$i\hbar\frac{\partial\Psi}{\partial t} = E\Psi \qquad .....(13)$$

Substituting equation (13) in Schrodinger time independent wave equation (11) we get,

$$\nabla^2\Psi + \frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\Psi}{\partial t} - V\Psi\right] = 0$$

or $$\nabla^2\Psi = \frac{-2m}{\hbar^2}\left[i\hbar\frac{\partial\Psi}{\partial t} - V\Psi\right]$$

Multiplying LHS and RHS by $-\frac{\hbar^2}{2m}$ and rearranging we get

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = \frac{i\hbar\,\partial\Psi}{\partial t} \qquad .....(14)$$

Equation (14) is known as Schrodinger time dependent wave equation.

Equation (14) can also be written as

$$(-\frac{\hbar^2}{2m}\nabla^2 + V)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

or $$H\Psi = E\Psi$$

Where $H = \left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)$ is called the **Hamiltonian operator** and $E = i\hbar\frac{\partial}{\partial t}$ is called the **energy operator.**

3.12 PHYSICAL SIGNIFICANCE OF A WAVE FUNCTION (Ψ)

1. Wave function Ψ must be finite everywhere.
2. Wave function Ψ is a complex quantity and tells the probability of finding the particle's position at the given time. Being a complex function it does not have a direct physical meaning.
3. A wave function Ψ must be a single valued.

4. Wave function Ψ must be a continuous and have a continuous first derivative every where.

5. $\Psi\Psi^* = |\Psi|^2$ is called the probability density and Ψ is also called probability amplitude.

6. If the particle is somewhere the integral of $|\Psi|^2 d\tau$ over the whole space must be unity $\int_{-\infty}^{\infty} |\Psi|^2 d\tau = 1$ where $d\tau = dx.dy.dz = dV$ represents a small volume. The wave functions that satisfy the above condition are known as normalized wave function.

3.13 APPLICATION OF SCHRODINGER WAVE EQUATION: PARTICLE IN A ONE DIMENSIONAL INFINITE DEEP POTENTIAL WELL

Let us consider a particle (electron, proton, etc.) restricted to move along the x-axis between x = 0 and x = L by ideally reflecting infinitely high potential well as shown in Fig. 3.10. Suppose the potential energy V of the particle is zero inside the potential well but rises infinitely on the outside then we have

$$V = 0 \text{ when } 0 < x < L$$

$$V = \infty \text{ when } 0 \geq x \geq L$$

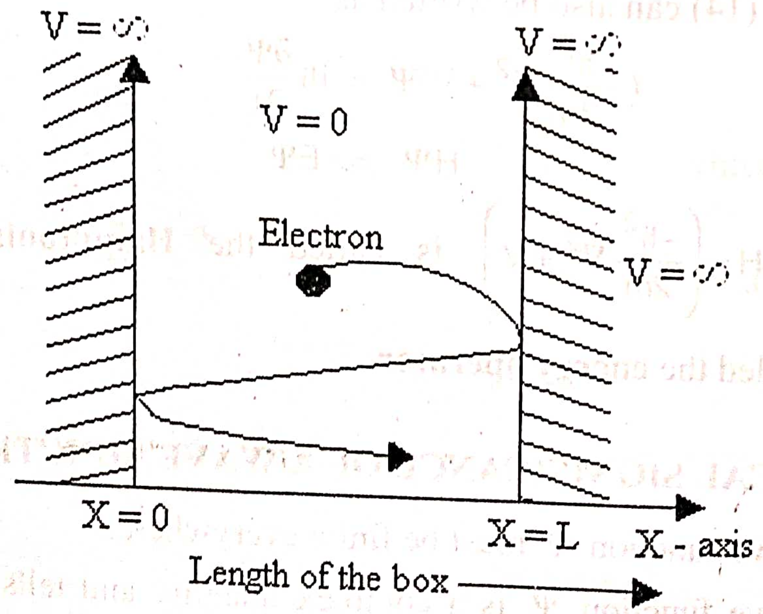

Fig. 3.10

In such a case the particle is said to be moving in an infinitely deep potential well.

The boundary condition for the wave function

$$\Psi = 0 \text{ for } x = 0 \qquad .....(1a)$$

$$\Psi = 0 \text{ for } x = L \qquad .....(1b)$$

We know that the Schrodinger one dimensional time independent wave equation

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\Psi = 0 \qquad .....(2)$$

The potential energy inside the box is zero (V = 0) then eqn. (2) becomes

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}E\Psi = 0 \qquad .....(3)$$

Let $k^2 = \frac{2mE}{\hbar^2}$ .....(4)

Then eqn. (3) can be rewritten as

$$\frac{d^2\Psi}{dx^2} + k^2\Psi = 0 \qquad .....(5)$$

The general solution for equation (5) is given by

$$\Psi(x) = A \sin kx + B \cos kx \qquad .....(6)$$

Where A and B are called arbitrary constants which can be calculated by applying boundary conditions.

Determination of A and B

Applying the first boundary condition (1a)

At $x = 0$, $\Psi = 0$

Then eqn. (6) becomes

$$0 = A \sin 0 + B \cos 0$$

$$0 = 0 + B(1)$$

$$B = 0 \qquad .....(7)$$

Applying the second boundary condition (1b)

At $x = L$, $\Psi = 0$

Then eqn. (6) becomes

$$0 = A \sin kL + B \cos kL \qquad .....(8)$$

Substituting eqn. (7) in (8) then

$$0 = A \sin kL \quad .....(9)$$

Here $A \neq 0$

$$\therefore \quad \sin kL = 0 \quad .....(10)$$

$$\therefore \quad kL = n\pi$$

or
$$k = \frac{n\pi}{L} \quad .....(11)$$

Substituting the value of B and k in equation (5) we can write the wave function of the free electron which is confined in a one dimensional box

$$\Psi = A \sin\frac{n\pi x}{L} \quad .....(12)$$

In general
$$\Psi_n = A \sin\left(\frac{\pi x}{L/n}\right)$$

To find Energy of the particle:

From equation (4)

$$k^2 = \frac{2mE}{\hbar^2}$$

$$= \frac{2mE}{\left(h^2/4\pi^2\right)} \qquad \text{Since } \hbar^2 = \frac{h^2}{4\pi^2}$$

or
$$k^2 = \frac{8\pi^2 mE}{h^2} \quad .....(13)$$

Squaring equation (11) we get

$$k^2 = \frac{n^2\pi^2}{L^2} \quad .....(14)$$

Equating equation (13) and equation (14)

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{L^2}$$

Energy of the particle (electron) $E = \frac{n^2h^2}{8mL^2}$ ....(15)

In general
$$E_n = n^2 . \left(\frac{h^2}{8mL^2}\right)$$

Where $n = 1,2,3,....$

From the above equation we can conclude that the energy values are discrete. The value of E_n is also called eigen value and Ψ_n is also called eigen function. The various energy levels and their wave functions of a particle enclosed in a one dimensional box are shown in the Fig. 3.11.

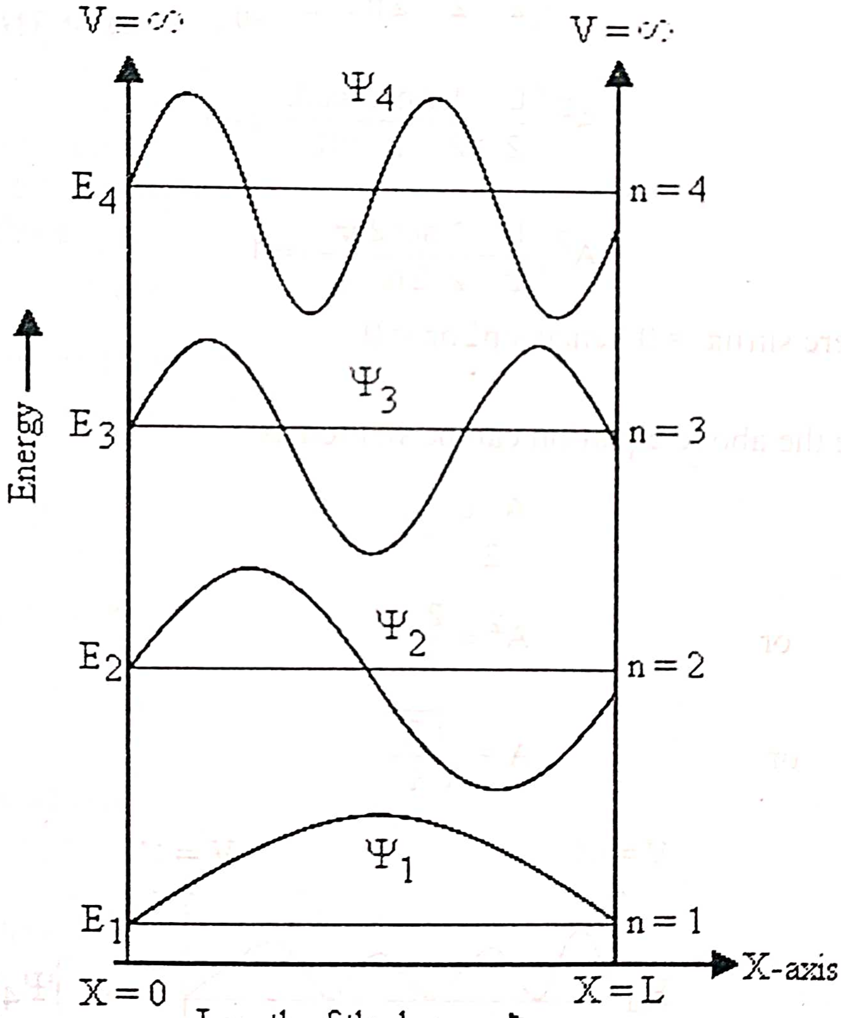

Fig. 3.11

Normalization of the wave function

For a one dimensional potential box of length 'L' the probability of finding the particle is given by

$$P = \int_0^L |\Psi|^2 \, dx = 1 \qquad .....(16)$$

Substituting equation (12) in equation (16), we get

$$P = \int_0^L A^2 \sin^2 \frac{n\pi x}{L} dx = 1$$

or $$A^2 \int_0^L \left(\frac{1-\cos 2n\pi x/L}{2} \right) dx = 1$$

$$A^2 \left(\frac{x}{2} - \frac{1}{2} \frac{\sin 2n\pi x/L}{2n\pi/L} \right)_0^L = 1$$

$$A^2 \left(\frac{L}{2} - \frac{1}{2} \frac{\sin 2n\pi L/L}{2n\pi/L} \right) = 1$$

$$A^2 \left(\frac{L}{2} - \frac{1}{2} \frac{\sin 2n\pi}{2n\pi/L} \right) = 1$$

Here $\sin n\pi = 0$ hence $\sin 2n\pi = 0$

Therefore the above equation can be written as

$$\frac{A^2 L}{2} = 1$$

or $$A^2 = \frac{2}{L}$$

or $$A = \sqrt{\frac{2}{L}} \quad \text{.....(17)}$$

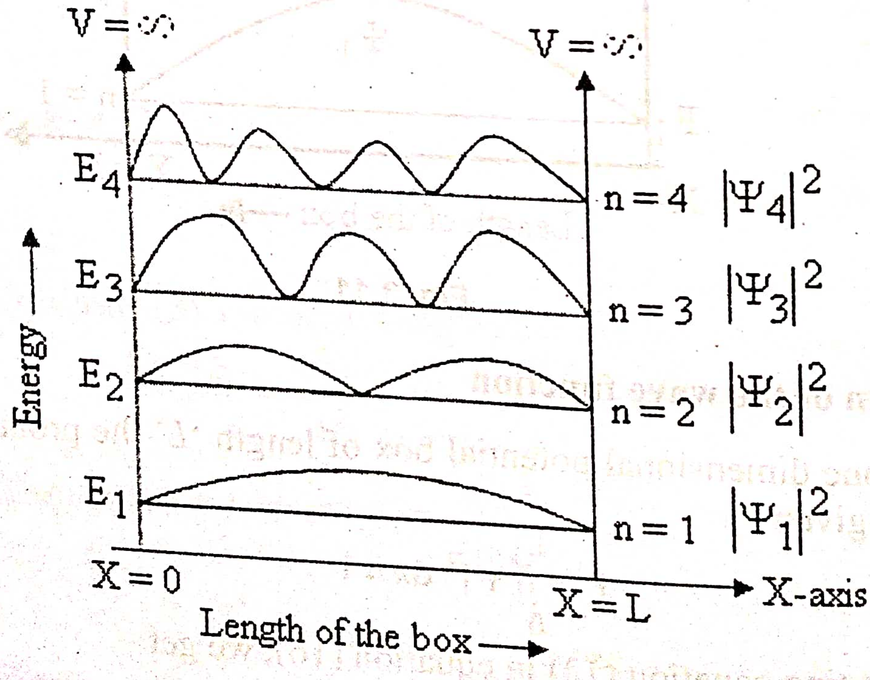

Fig. 3.12

Substituting Eqn (17) in eqn. (12) we get the normalized wave function

$$\Psi_n = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L} \qquad .....(18)$$

The normalized wave function and their energy values are shown in Fig. 3.12.

3.14 THREE DIMENSIONAL POTENTIAL BOX

The solution of one-dimensional potential well can be extended for a three dimensional potential well. In a three dimensional potential well the particles (electron, proton, etc.) can move in random direction. Instead of using only one quantum number, three quantum numbers (n_x, n_y and n_z) corresponding to the three co-ordinate axis (x,y and z) are introduced.

If a,b,c are the length of the box along x,y and z axis then the energy of the particle $E = E_x + E_y + E_z$. In general

$$E_{n_x,n_y,n_z} = \frac{n_x^2h^2}{8\,m\,a^2} + \frac{n_y^2h^2}{8\,m\,b^2} + \frac{n_z^2h^2}{8\,mc^2}$$

If a = b = c for a cubical box, energy eigen values can be written as

$$E_{n_x,n_y,n_z} = \frac{h^2}{8\,m\,a^2}\left(n_x^2 + n_y^2 + n_z^2\right) \qquad ....(1)$$

The corresponding normalized wave function of an electron in a cubical box can be written as

$$\Psi_{n_x,n_y,n_z} = \sqrt{\frac{2}{a}\times\frac{2}{a}\times\frac{2}{a}}\,.\sin\frac{n_x\pi x}{a}\sin\frac{n_y\pi y}{a}\sin\frac{n_z\pi z}{a}$$

$$\therefore \Psi_{n_x,n_y,n_z} = \sqrt{\frac{8}{a^3}}\,.\sin\frac{n_x\pi x}{a}\sin\frac{n_y\pi y}{a}\sin\frac{n_z\pi z}{a} \qquad .....(2)$$

3.15 DEGENERACY AND NON – DEGENERACY

Degenerate state

When the three independent stationary states having quantum numbers such as (2 3 3), (3 2 3), (3 3 2) for n_x, n_y and n_z have the same energy value $22.\left(\frac{h^2}{8mL^2}\right)$. Hence E_{233}, E_{323}, E_{332} and the corresponding wavefunction Ψ_{233}, Ψ_{323}, Ψ_{332} are called degenerate. Hence different combination of quantum numbers gives the same eigen value with different eigen functions are called degenerate state. The degenerate state will be affected by electric and magnetic field.

Non-Degenerate state

If the various combinations of quantum number describes the same energy eigen value and same eigen function then such state and energy level are said to be non-degenerate state.

Example:

For $n_x = 3;\ n_y = 3;\ n_z = 3$

$$E_{333} = \frac{27h^2}{8ma^2} \text{ and } \Psi_{333} = \sqrt{\frac{8}{a^3}} \sin\frac{3\pi x}{a} \sin\frac{3\pi y}{a} \sin\frac{3\pi z}{a}$$

3.16 MICROSCOPE

Microscope is an instrument used to view the magnified image of a smaller object. Generally microscopes are classified into simple and compound microscopes. In a simple microscope only one lens is used but in compound microscope two or more lenses are used.

Further they are classified into metallurgical microscope, electron microscope, ultraviolet microscope, etc.

Basic definitions of microscope

(i) Magnifying Power

The magnifying power (M) of a microscope is defined as the ratio between the angles subtended by the final image at the eye to the angle subtended by the object at eye placed at the near point.

$$M = \frac{\text{Angle subtended by the final image at eye } (\beta)}{\text{Angle subtended by the object at eye placed at the near point } (\alpha)}$$

$$M = \frac{\beta}{\alpha}$$

(ii) Resolving Power

It is the ability of an optical instrument to form distinct and separable images of the two point objects which are close to each other.

If '*d*' is the least distance between two close point objects then we can write

$$d = \frac{\lambda}{2\ N.A}$$

$$\text{Resolving power } \frac{1}{d} = \frac{2\ N.A}{\lambda}$$

Where N.A. is the Numerical Aperture of the microscope and λ is the wavelength of light through vacuum.

An optical microscope can resolve only a few hundreds of nanometer separation and the magnification is about 2000x.

3.17 ELECTRON MICROSCOPE

It is an instrument that uses highly energetic electron beam to examine a very small specimen.

Principle: The high energy electron beam is allowed to fall over the specimen and image formed due to the transmitted electron beam from the specimen is examined.

Construction

Essential parts of the electron microscope

i) An electron source
ii) Electro magnetic lenses
iii) Metal aperture
iv) Object holder
v) Screen

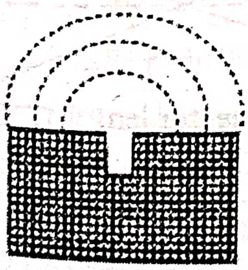

Fig. 3.13

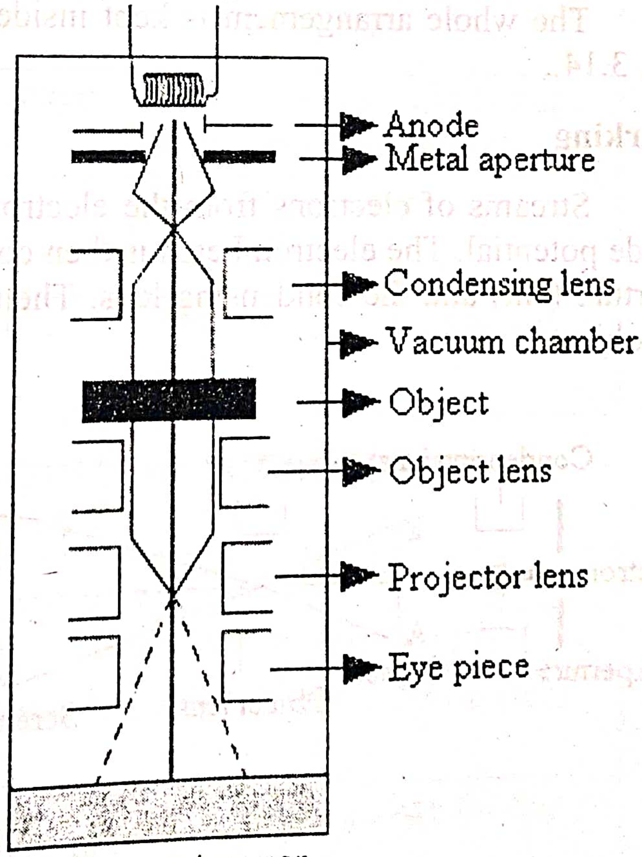

Fig. 3.14

Description

Electron gun is made of tungsten filament. Electrons which are emitted due to thermionic emission by the filament are accelerated by a large potential applied to the electrodes of the electron gun.

Electro magnetic lenses are made of coils enclosed inside the iron shield which has a gap at the middle as shown in the Fig. 3.13. If the gaps of the two electromagnetic lenses are faced with each other uniform magnetic field is produced. Similarly if the gaps of the two electromagnetic lenses are slightly disturbed non-uniform magnetic field is produced. Electron beam can be focused by the electromagnetic lens.

In this system we have three magnetic lenses.

i) Condensing lens which is used to condense the electron beam.

ii) Objective lens which is used to resolve the structures of the object.

iii) Projector lens which is used to enlarge the object.

Metal aperture is used to get a narrow beam and object holder holds the object. Enlarged image of the object is seen through the fluorescent screen.

The whole arrangement is kept inside a vacuum chamber as shown in the Fig. 3.14.

Working

Streams of electrons from the electron gun are accelerated by the positive anode potential. The electron beam is then confined to a narrow beam by the metal aperture (slit) and the condensing lens. Then the electron beam is passed through the object.

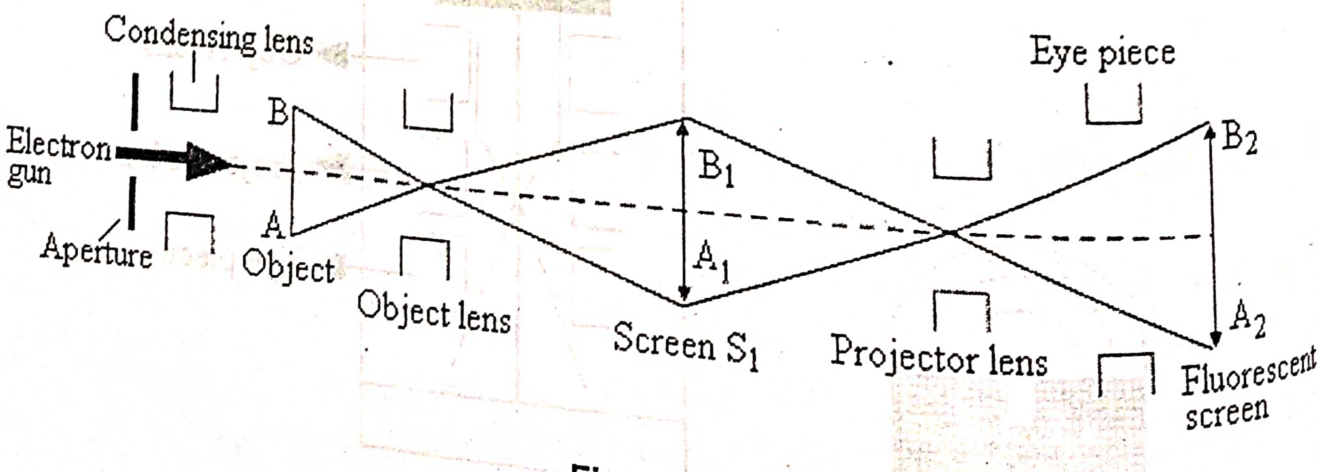

Fig. 3.15

An interaction between the electron beam and the object occurs and the transmitted electron beam carries the image of the object. Then it is passed through the magnifying objective lens as shown in Fig. 3.15. This lens magnifies the images of the object more than 100 times. Then the image is made to fall on the screen S_1 and the electron beam is passed through the magnifying projector lens. It also magnifies the image of the object again more than 10 times. Finally the image of the object is made to fall on a fluorescent screen. The image formed on the fluorescent screen is viewed through an optical lens which is attached with the eyepiece. It also magnifies the image 10 times. Therefore total magnification in the order of more than 10^5 times is achieved.

Merits

i) The magnification is 100000X.

ii) Focal length of the microscope can be varied.

Applications

i) It is used to determine the complicated structure of the crystal.

ii) It is used to study the disease due to virus and bacteria.

iii) It is used to study and analysis of colloidal particles.

iv) It is used to study the composition of papers, paints etc.

3.18 DIFFERENCES BETWEEN TELESCOPE AND MICROSCOPE

S.No.	Telescope	Microscope
1	We get the magnified image of the distant object.	We get the magnified image of the very small object.
2	Eye piece is small.	Eye piece is large.
3	Object is large.	Object is small.
4	Permanent record is not possible.	Permanent record is possible

3.19 DIFFERENCES BETWEEN OPTICAL MICROSCOPE AND ELECTRON MICROSCOPE

S.No.	Optical microscope	Electron microscope
1	Light source is used	Source is an electron gun
2	Optical lens system is used	Electro magnetic lens system is used
3	Vacuum is not necessary for the operation.	Vacuum is necessary for the operation.
4	Magnification is 2000 X	Magnification is 100000 X

1. **An electron is accelerated by a potential difference of 140 V. What is the wave length of the electron?**

Solution :

Given: Accelerated voltage of the electron V = 140 V

Formula: de Broglie wavelength $\lambda = \frac{h}{\sqrt{2meV}}$

$$\therefore \lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 140}}$$

(or) $$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{4.08128 \times 10^{-47}}}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{6.3884 \times 10^{-24}}$$

(or) $$\lambda = 1.0370 \times 10^{-10} \text{ metres}$$

The De-Broglie wavelength = $\lambda = 1.0370$ Å

2. **An electron at rest is accelerated through a potential of 4000V. Calculate the de Broglie wavelength of matter wave associated with it.**

Solution :

Given: V = 4000 V

Formula: de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2meV}}$

$$\therefore \lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 4000}}$$

(or) $$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{1.1661 \times 10^{-45}}}$$ or $$\lambda = \frac{6.625 \times 10^{-34}}{3.4148 \times 10^{-23}}$$

$$\lambda = 1.940 \times 10^{-11} \text{ m (or) } 0.1940 \text{ Å}$$

De-Broglie wavelength of electron = 0.1940 Å

3. Calculate the de-Broglie wave length of an electron of energy 150eV.

Solution :

Given : Energy of electron $E = 150$ eV

or $E = 150 \times 1.6 \times 10^{-19}$ Joules

or $E = 2.4 \times 10^{-17}$ Joules

Formula :

We know that de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

Substituting the given values, we have

$$\therefore \lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 2.4 \times 10^{-17}}}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{4.3728 \times 10^{-47}}}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{6.6127 \times 10^{-24}}$$

$$\lambda = 1.0019 \times 10^{-10} \text{ metres}$$

De-Broglie wavelength $\lambda = 1.0019$ Å

4. **Calculate the de-Broglie wave length associated with a porton moving with a velocity equal to 1/10 th of the velocity of light. Mass of proton = 1.675×10^{-27} kg.**

Solution :

Given : Mass of the proton $m = 1.675 \times 10^{-27}$ kg

Velocity of proton $v = \frac{1}{10} \times$ velocity of light,

$$v = \frac{1}{10} \times 3 \times 10^{8}$$

Velocity $v = 30 \times 10^{6}$ m/s

Formula : de-Broglie wavelength $\lambda = \frac{h}{mv}$

$$\lambda = \frac{6.625 \times 10^{-34}}{1.675 \times 10^{-27} \times 30 \times 10^{6}} = 1.3184 \times 10^{-14} \text{ m}$$

De-Broglie wavelength $\lambda = 1.3184 \times 10^{-14}$ m

5. **A neutron of mass 1.675×10^{-27} kg is moving with a kinetic energy 15 KeV. Calculate the de-Broglie wavelength associated with it.**

Solution :

Given : Energy of neutron $E = 15$ KeV

$$E = 15 \times 10^{3} \times 1.6 \times 10^{-19} \text{ Joules}$$

$$E = 2.4 \times 10^{-15} \text{ Joules}$$

Formula : de-Broglie wavelength $\lambda = \dfrac{h}{\sqrt{2mE}}$

$$\therefore \lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 2.4 \times 10^{-15}}}$$

(or) $$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{8.04 \times 10^{-42}}}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{2.8355 \times 10^{-21}}$$

$$\therefore \lambda = 2.3365 \times 10^{-13} \text{ m}$$

De-Broglie wavelength of neutron = 2.3365×10^{-13} m

6. **Calculate the energy of the electron in the energy level immediately after the lowest energy level, confined in a cubical box of side 1 nm. Also find the temperature at which the average energy of the molecules of a perfect gas would be equal to the energy of the electron in the above said level.**

Solution :

4 For a cubical box the energy eigen value is

$$E_{n_x, n_y, n_z} = \frac{h^2}{8\,m\,L^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

For the next energy level to the lowest energy level $n_x = 1$ $n_y = 1$ and $n_z = 2$.

$$\therefore E_{112} = \frac{h^2}{8\,m\,L^2}\left(1^2 + 1^2 + 2^2\right)$$

$$\therefore E_{112} = \frac{6h^2}{8mL^2}$$

Given $L = 1 \times 10^{-9}$ m

$m = 9.11 \times 10^{-31}$ kg

$h = 6.625 \times 10^{-34}$ J sec.

$$\therefore E_{112} = \frac{6 \times (6.625 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (1 \times 10^{-9})^2}$$

$$= \frac{2.6334 \times 10^{-66}}{7.288 \times 10^{-48}}$$

$$\mathbf{E_{112} = 3.6133 \times 10^{-19}\ Joules}$$

ii) We know the average energy of the molecules of a perfect gas $= \frac{3}{2} K_B T$

$$\therefore E_{112} = \frac{3}{2} K_B T$$

(or) The temperature $T = \frac{2E_{112}}{3K_B}$

$$= \frac{2 \times 3.6133 \times 10^{-17}}{3 \times 1.38 \times 10^{-23}}$$

Temperature of the molecules $T = 1.7456 \times 10^4$ K

7. **Calculate the minimum energy an electron which possess in an infinitely deep potential well of width 10 nm.**

Solution :

Given: $L = 10$ nm $= 10 \times 10^{-9}$ m

We know for minimum energy $n = 1$

Formula: Energy of an electron in an infinitely deep potential well

$$E = \frac{n^2h^2}{8mL^2}$$

$$\therefore E = \frac{1^2 \times (6.625 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (10 \times 10^{-9})^2}$$

(or) $$E = \frac{4.3891\times10^{-67}}{7.288\times10^{-46}}$$

$$E = 6.0223 \times 10^{-22}\ J$$

(or) $$E = \frac{6.0223\times10^{-22}}{1.6\times10^{-19}}\ eV$$

$$E = 3.764 \times 10^{-3}\ eV$$

Minimum energy of an electron (E) = 3.764×10^{-3} eV

8. **Find the energy of an electron moving in one dimension in an infinitely high potential box of width 0.2nm**

Solution :

Given: Length (or) width of one dimension box $L = 0.2\ nm = 0.2\times10^{-9}\ m$

Formula: The energy of an electron is $E = \frac{n^2h^2}{8mL^2}$

For Lowest energy n = 1.

$$\therefore E = \frac{1^2h^2}{8mL^2}$$

$$\therefore E = \frac{(6.625\times10^{-34})^2}{8\times9.11\times10^{-31}\times(0.2\times10^{-9})^2}$$

(or) $$E = 1.5056 \times 10^{-18}\ J$$

(or) $$E = \frac{1.5056\times10^{-18}}{1.6\times10^{-19}}\ eV$$

Energy of the electron E = 9.4098 eV

9. **Calculate the number of photons emitted by a 150 watts sodium vapour lamp. [Given λ = 5893 Å]**

Solution :

Given: Power = 150 Watts

Formula : $\text{Energy} = h\nu = \frac{hc}{\lambda}$

$$\therefore \text{Energy } E = \frac{6.625\times10^{-34}\times3\times10^{8}}{5893\times10^{-10}}$$

$$= 3.3726 \times 10^{-19}\ \text{Joules}$$

$$\text{Number of photons emitted} = \frac{\text{Power}}{\text{Energy}}$$

$$= \frac{150\ J/S}{3.3726 \times 10^{-19}\ \text{Joules}}$$

$$= 4.4475 \times 10^{20} \text{ per second}$$

Number of photons emitted = 4.4475×10^{20}

10. An electron is confined to a one-dimensional box of side 10^{-10} m. Obtain the first four Eigen values of the electron.

Solution :

Given: Sides of the box $L = 10^{-10}$m

Formula : Energy eigen value $E = \frac{n^2h^2}{8mL^2}$

First Eigen value $E_1 = \frac{1^2 \times (6.625 \times 10^{-34})^2}{8 \times (9.11 \times 10^{-31})(10^{-10})^2}$

$$E_1 = \frac{4.38906 \times 10^{-67}}{7.288 \times 10^{-50}}$$

$$E_1 = 6.022 \times 10^{-18} \text{ J}$$

(or) $E_1 = \frac{6.022 \times 10^{-18}}{1.6 \times 10^{-19}} = 37.6395 \text{ eV}$

Second Eigen value

$$E_2 = 2^2 E_1 = 2.4089 \times 10^{-17}\text{J (or) } 150 \text{ eV}$$

Third Eigen value

$$E_3 = 3^2 E_1 = 5.4198 \times 10^{-17}\text{J (or) } 338 \text{ eV}$$

Fourth Eigen value

$$E_4 = 4^2 E_1 = 9.6352 \times 10^{-17}\text{J (or) } 602 \text{ eV}$$

11. Calculate the lowest energy of the system containing two electrons confined to a box of length 1 Å.

Solution :

Given: $L = 1$Å

Formula : Energy of the system having two electrons

$$E = 2\left(\frac{n^2h^2}{8m\,l^2}\right)$$

$$\therefore E = 2\left(\frac{(1)^2 \times (6.625\times10^{-34})^2}{8\times9.11\times10^{-31}\times(1\times10^{-10})^2}\right)$$

$$E = 1.2045 \times 10^{-17} \text{ Joules}$$

or $$E = \frac{1.2044\times10^{-17}}{1.6\times10^{-19}} \text{eV}$$

$$\mathbf{E = 75.279\ eV}$$

Energy of the system having two electrons = 75.279 eV

SHORT QUESTIONS AND ANSWERS

1. What is quantum Physics?

Quantum Mechanics is the theoretical basis of Modern Physics that explains the nature and behavior of matter and energy on the atomic and subatomic level.

Quantum Mechanics (also called wave mechanics) deals with the investigation of the behavior of micro-particles. Most advances that have taken place in Solid State Physics, Atomic Physics and Nuclear Physics are based on the principles of Quantum Mechanics.

2. What is meant by black body radiation?

A perfect black body is one which absorbs all the thermal radiations incident upon it and it does not reflect light. There is no perfect black body exist. An object coated with a black pigment is nearly a black body. The light emitted by a black body is called black-body radiation.

3. State Wein's displacement law.

This law states that the product of the wavelength corresponding to maximum energy and the absolute temperature is a constant.

$$\lambda_m T = \text{constant}$$

4. State Rayleigh – Jean's law.

The emissive power of the black body at absolute temperature T and at a given wavelength λ is directly proportional to T and inversely proportional to λ^4.

$$E_\lambda = \frac{8\pi K_B T}{\lambda^4}$$

5. List out the characteristics of photon.

i. Max Planck introduced the concepts that emission or absorption of electromagnetic radiation takes place as discrete quanta or tiny discrete packets called 'photons' each having an energy hv where h is the Planck's constant and v is the frequency of radiation.

ii. Photons are not affected by electric and magnetic field and they are electrically neutral.

iii. Mass of the photon $m = h/c\lambda$ and momentum of the photon $P = h/\lambda$ since $P = mc$.

iv. Velocity of the photon is 3×10^8 m/sec and it is equal to the velocity of light.

6. What is Compton effect and Compton shift?

When a monochromatic beam of x-rays is scattered by a substance, the scattered x-rays contain radiation not only of the same wavelength as that of unmodified primary radiation but also the modified radiation of longer wavelength. This is called 'Compton effect'. The difference between scattered wavelengths is called 'Compton shift'.

7. What is meant by matter waves or De-Broglie waves?

In 1924, Louis Victor due de Broglie a French scientist proposed that light waves behaves sometimes as particles and some other times behaves as waves. Hence the material particles like atoms, molecules, electrons, protons or neutrons behave as waves are known as matter waves or de Broglie waves. This concept of particle-wave duality is called De-Broglie hypothesis.

8. List the properties of matter waves.

i. Greater the mass of the particle smaller will be the wavelength of the matter waves.

ii. Matter waves are not electromagnetic waves.

iii. Matter waves can travel faster than light.

9. What is Schrodinger wave equation?

Schrodinger equation is one of the basic equations in quantum mechanics. This equation can be applied for both macroscopic and microscopic particles. Schrodinger derived a mathematical equation to describe the dual nature of matter waves. The equation describes the wave nature of a particle in mathematical form is known as Schrodinger wave equation.

10. Write down the Schrodinger wave equations.

There are two types of Schrodinger wave equations viz.

(i) Schrodinger three dimensional time dependent wave equation is

$$(-\frac{\hbar^2}{2m}\nabla^2 + V)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

Or $$E\Psi = H\Psi$$

Where E $\rightarrow$ Total energy of the particle

H $\rightarrow$ Hamiltonian operator

Ψ $\rightarrow$ Wave function

(ii) Schrodinger three dimensional time independent wave equation is

$$\Delta^2\Psi + \frac{2m}{\hbar^2} = [E - v]\Psi = 0 \quad \text{(3 dimensional)}$$

Where E $\rightarrow$ Total energy of the particle

V $\rightarrow$ Potential energy of the particle

m $\rightarrow$ mass of the particle

$\hbar \rightarrow \frac{h}{2\pi}$ (h $\rightarrow$ Planck's constant)

11. What are the physical significances of wave function?

1. Wave function Ψ must be finite everywhere.
2. Wave function Ψ is a complex quantity and tells the probability of finding the particle's position at the given time. Being a complex function it does not have a direct physical meaning.
3. A wave function Ψ must be a single valued.
4. Wave function Ψ must be a continuous and have a continuous first derivative every where.
5. $\Psi\Psi^* = |\Psi|^2$ is called the probability density and Ψ is also called probability amplitude.
6. If the particle is somewhere the integral of $|\Psi|^2 d\tau$ over the whole space must be unity $\int_{-\infty}^{\infty} |\Psi|^2 d\tau = 1$ where $d\tau = dx.dy.dz = dV$ represents a small volume. The wave functions that satisfy the above condition are known as normalized wave function.

12. What to you understand by the term wave function?

Wave function (Ψ) is a variable quantity that is associated with a moving particle at any position (x,y,z) and at any time 't'.

13. Define normalization process and write down the normalized wave function for an electron in a one dimensional potential well of length 'a' meters.

Normalization is the process by which the probability of finding a particle inside any potential well can be done.

For a one dimensional potential well of length 'a' meter the normalized wave function is given by

$$\Psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

14. Write the formula for finding the Eigen value and Eigen function.

Eigen value is defined as energy of the particle and is denoted by the letter (E_n).

$$E_n = \frac{n^2h^2}{8ml^2}$$

Eigen function is defined as the wave function of the particle and is denoted by the letter (Ψ_n).

$$\Psi_n = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a}$$

15. What is degenerate and non-degenerate state? Give examples.

When the three independent stationary states having quantum numbers such as (2 3 3), (3 2 3), (3 3 2) for n_x, n_y and n_z have the same energy value $22.\left(\frac{h^2}{8mL^2}\right)$. Hence E_{233}, E_{323}, E_{332} and the corresponding wavefunction Ψ_{233}, Ψ_{323}, Ψ_{332} are called degenerate. Hence different combination of quantum numbers gives the same eigen value with different eigen functions are called degenerate state. The degenerate state will be affected by electric and magnetic field.

If the various combinations of quantum number describes the same energy eigen value and same eigen function then such state and energy level are said to be non-degenerate state.

Example:

For $n_x = 3;\ n_y = 3;\ n_z = 3$

$$E_{333} = \frac{27h^2}{8ma^2} \text{ and } \Psi_{333} = \sqrt{\frac{8}{a^3}} \sin\frac{3\pi x}{a} \sin\frac{3\pi y}{a} \sin\frac{3\pi z}{a}.$$

16. Define magnifying power.

The magnifying power (M) of a microscope is defined as the ratio between the angles subtended by the final image at the eye to the angle subtended by the object at eye placed at the near point.

$$M = \frac{\text{Angle subtended by the final image at eye (b)}}{\text{Angle subtended by the object at eye placed at the near point (a)}}$$

$$M = \frac{\beta}{\alpha}$$