SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

VECTOR FIELDS & VECTOR CALCULUS

What We'll Discuss

TOPIC OUTLINE

Vector Fields
Vector Calculus
1. Vector addition
2. Vector Multiplication
    a.dot product
    b. cross product

Scalar and Vector fields:

Fields are classified as

1. Scalar field –Scalars are quantities characterized by magnitude only and algebraic sum.

    Examples : atmospheric temperature and Pressure

2. Vector field - magnitude and direction

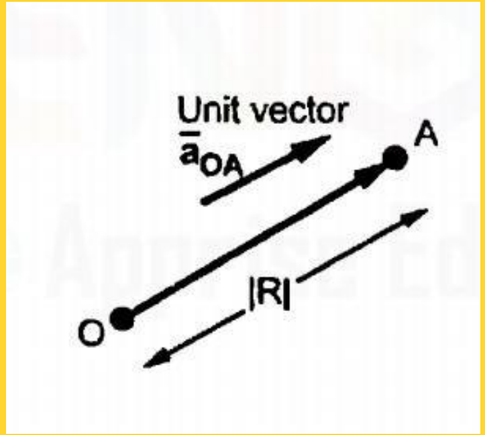
    Examples : wind velocity and gravitational force in atmosphere.

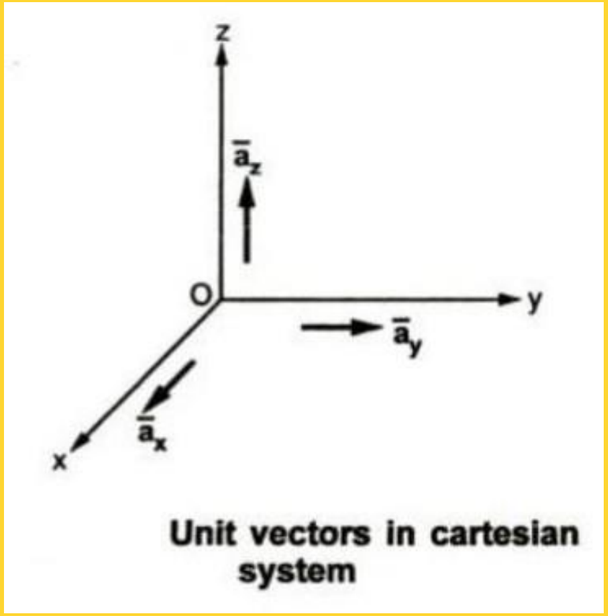
Vectors:

- Vectors are represented by “arrows”.
- Vectors have a “magnitude,” or length, and a direction.
- A vector can be expressed in terms of these two properties: the length of the vector and direction

Unit Vectors:

- A **unit vector** is a vector with a magnitude of one unit.
- Any vector can be expressed as a scalar multiple of its unit vector.



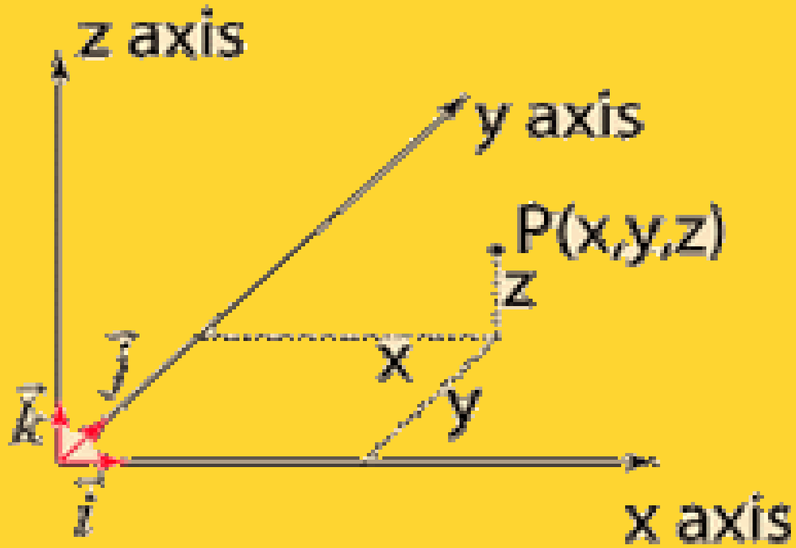


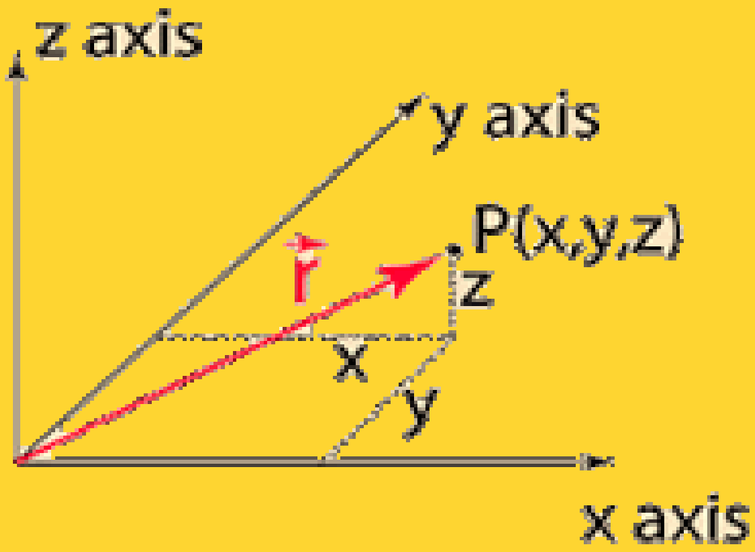
Unit vectors in cartesian system

$$\text{Unit vector } \bar{a}_{OA} = \frac{\overline{OA}}{|\overline{OA}|}$$

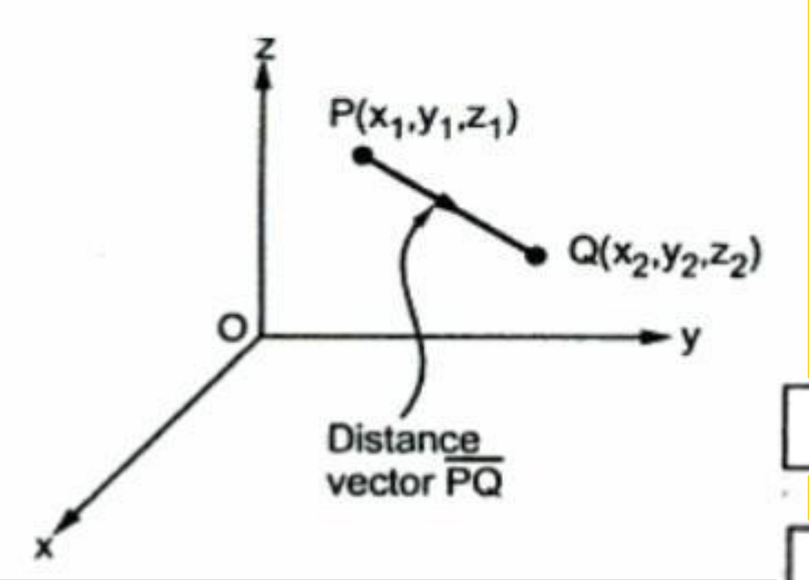
Position Vectors:

Position vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$





Distance Vectors:



$$\bar{P} = x_1 \bar{a}_x + y_1 \bar{a}_y + z_1 \bar{a}_z$$

$$\bar{Q} = x_2 \bar{a}_x + y_2 \bar{a}_y + z_2 \bar{a}_z$$

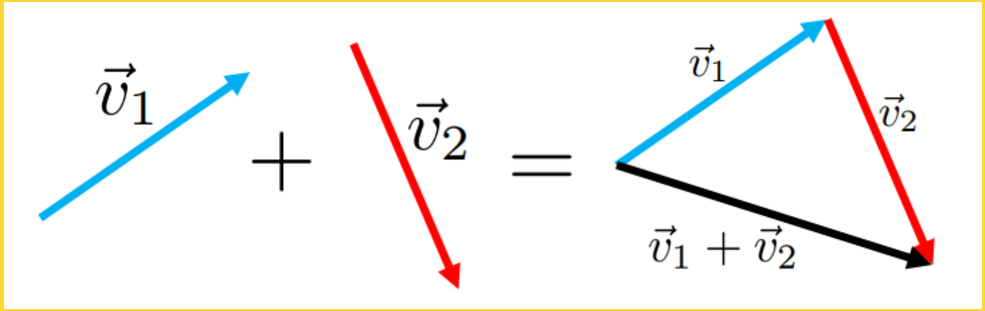
$$\overline{PQ} = \bar{Q} - \bar{P} = [x_2 - x_1]\,\bar{a}_x + [y_2 - y_1]\,\bar{a}_y + [z_2 - z_1]\,\bar{a}_z$$

$$|\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\bar{a}_{PQ} = \text{Unit vector along PQ} = \frac{\overline{PQ}}{|\overline{PQ}|}$$

Vector Calculus:

1. Vector Addition



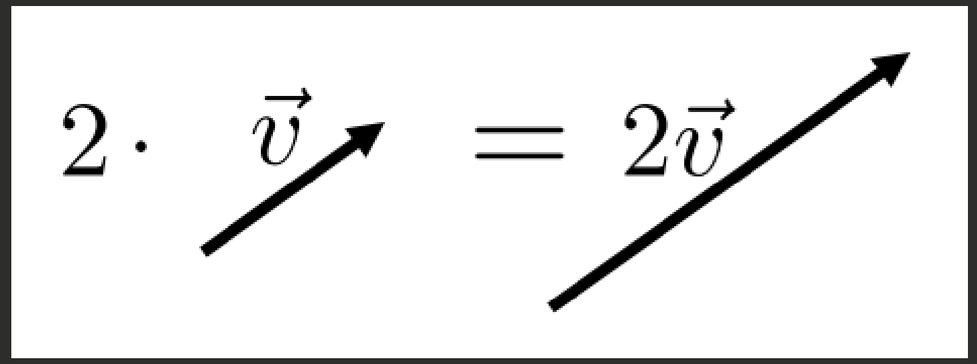
Vector Multiplication

- There are several different processes that can be described as “vector multiplication.”
- First, there is scalar multiplication.
- There are also ways to “multiply” two vectors together.
- There are two products:

  the **dot product** and the **cross product**.

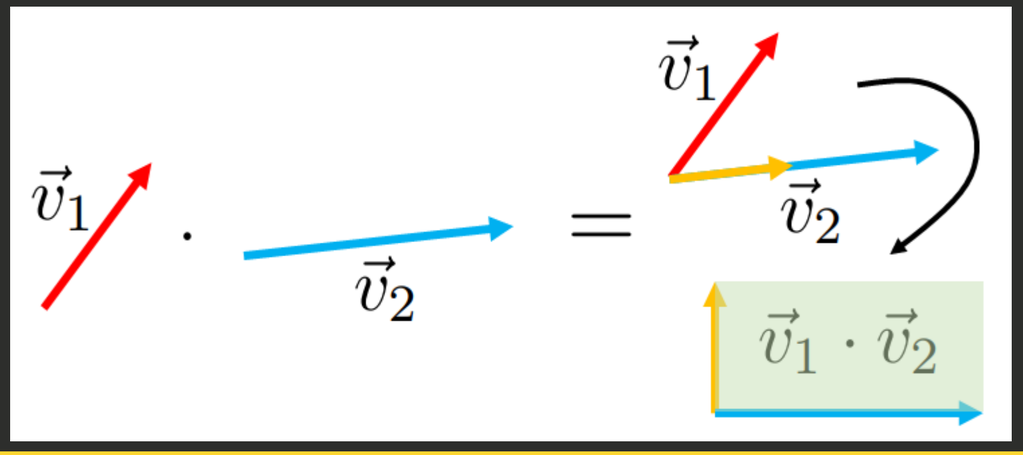
1. Scalar Multiplication

- Basically, when a vector is multiplied by a scalar i.e., a number, its length gets multiplied by that number(scalar).



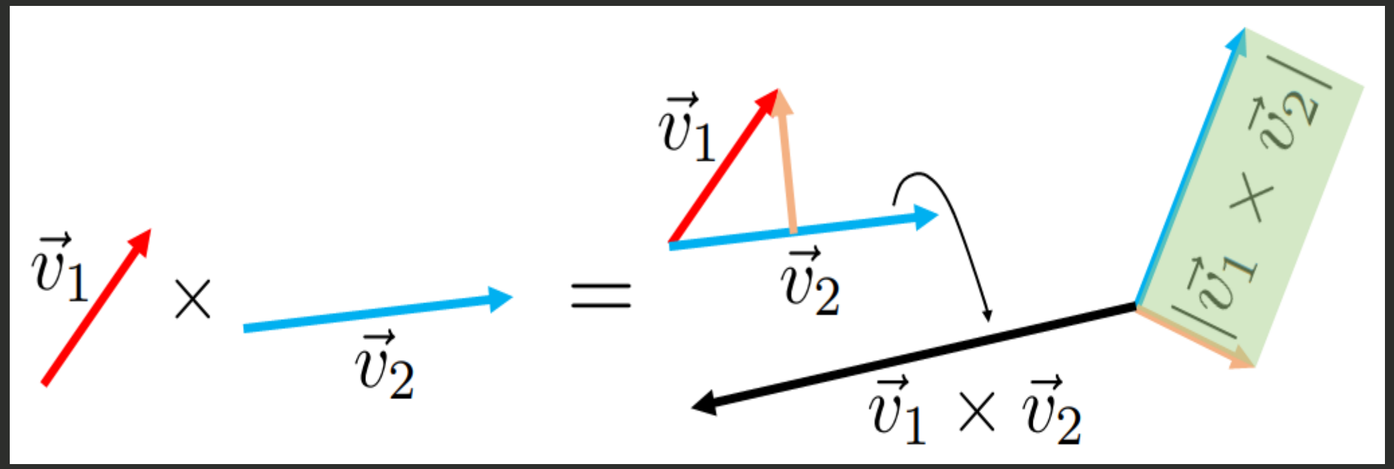
2a. The Dot Product

- The **dot product** is an operator between two vectors that returns a scalar. Specifically, a dot product takes the first vector's "component" along the direction of the second vector. It then takes the length of that component and multiplies it by the length of the second vector



2b. The Cross Product

- The **cross product** is an operator between two vectors that returns another vector. Specifically, it takes the length of the component of the first vector in the direction *perpendicular* to the direction of the second vector. It then returns a vector with the length of that first component multiplied by the length of the second vector. The direction of the cross product is given by the right-hand rule (by convention)



THANK YOU