LAGRANGE'S THEOREM:

Let G be a finite group of order 'n' & H be any subgroup of G. Then the order of H divides the order of G, (ie) $O(H) / O(G)$
(or) The order of each subgroup of a finite group is a divisor of the order of the group.

Proof: Let $(G,*)$ be a group whose order is n.

(ie) $O(G) = n$.

Let $(H,*)$ be a subgroup of G whose order is m.

(ie) $O(H) = m$

Let $h_1, h_2, h_3, \dots, h_m$ be the 'm' different elements of H.

The right coset $H*a$ of H in G is defined by

$H*a = \{h_1*a, h_2*a, \dots, h_m*a\}, \ a \in G$

Since there is a one-one correspondence btwn. the elts. of H and $H*a$, the elts of $H*a$ are distinct.

Hence each right coset of H in G has 'm' distinct elts.

WKT any right cosets of H in G are either disjoint or identical

The no. of distinct right cosets of H in G ~~are~~ either is finite (say k) [$\because$ G is finite]

The union of these k distinct cosets of H in G is equal to G.

Let these k distinct right cosets be

$$H*a_1,\ H*a_2,\ H*a_3 \dots,\ H*a_k$$

then $G = (H*a_1) \cup (H*a_2) \cup \dots \cup (H*a_k)$

$\therefore\ O(G) = O(H*a_1) + O(H*a_2) + \dots + O(H*a_k)$

$n = m + m + \dots + m$ (k times)

$n = km$

$\Rightarrow k = \frac{n}{m}$ (ie) $\frac{O(G)}{O(H)} = k$

Since k is an integer (time), m is a divisor of n.

$\Rightarrow O(H)$ is a divisor of $O(G)$

$\Rightarrow O(H)$ divides $O(G)$.