Algebraic Structures

Algebraic System:

A non empty set G together with one or more binary operations is called an algebraic system. We denote it by [G, *]

Note:
+, −, ·, ×, *, ∪, ∩, etc are some of binary operations.

Groups:

A non empty set G with the binary operator * i.e., (G, *) is said to be group, if it satisfies the following conditions.

1). closure property:
$\forall a, b \in G, \; a * b \in G$

2). Associative property:
$\forall a, b, c \in G, \; (a*b)*c = a*(b*c)$

3). Identity Element:
$\forall a \in G, \exists \, e \in G$ such that $a * e = e * a = a$ where e is the identity element.

4). Inverse Element:
$\forall a \in G, \exists \, a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$ where a^{-1} is the inverse element.

5). commutative property:
$\forall a, b \in G, \; a*b = b*a$ is called Abelian group.

Example:

for all a, b, c ∈ G	(G, +)	(G, ×)
1). closure	$a+b \in G$	$ab \in G$
2). Associative	$(a+b)+c = a+(b+c)$	$(a\times b)\times c = a\times(b\times c)$
3). Identity	$a+0 = 0+a = a$ '0' is the Additive Identity	$a\times 1 = 1\times a = a$ '1' is the multiplicative Identity
4). Inverse	$a+(-a) = (-a)+a = 0$ '$-a$' is the Additive inverse	$a\times a^{-1} = a^{-1}\times a = 1$ 'a^{-1}' is the multiplicative inverse
5). Commutative	$a+b = b+a$	$a\times b = b\times a$