Differential Calculus

1. $\frac{d}{dx}(C) = 0$, C is a constant
2. $\frac{d}{dx} x^n = nx^{n-1}$
3. $\frac{d}{dx} e^x = e^x$
4. $\frac{d}{dx} \log x = \frac{1}{x}$
5. $\frac{d}{dx} \sin x = \cos x$
6. $\frac{d}{dx} \cos x = -\sin x$
7. $\frac{d}{dx} \tan x = \sec^2 x$
8. $\frac{d}{dx} \sec x = \sec x \tan x$
9. $\frac{d}{dx} \csc x = -\csc x \cot x$
10. $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$
11. $\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}$
12. $\frac{d}{dx}\left[\frac{-1}{x^2}\right] = \frac{2}{x^3}$
13. $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$
14. $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
15. $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$
16. $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
17. $\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$
18. $\frac{d}{dx} \operatorname{cosec}^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$
19. $\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$

Hyperbolic functions:

$\sinh x = \frac{e^x - e^{-x}}{2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

20. $\frac{d}{dx} \sinh x = \cosh x$
21. $\frac{d}{dx} \cosh x = \sinh x$
22. $\frac{d}{dx}(uv) = u\,dv + v\,du$
23. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$

$\frac{\text{anything}}{0} = \infty$

$\frac{0}{\text{anything}} = 0$

$e^{-\infty} = 0, \quad e^0 = 1$

Curvature:

The rate of bending of a curve at any point on it is called the curvature of the curve at that point.

Radius of Curvature:

The reciprocal of the curvature of the curve at any point is called the radius of curvature at that point. It is denoted by ρ.

Formula:

Let $y = f(x)$ be the given curve. Then

$$\rho = \frac{\left[1+\left[\frac{dy}{dx}\right]^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

If $\frac{dy}{dx} = \infty$ at a point on the curve $y = f(x)$, then

$$\rho = \frac{\left[1+\left(\frac{dx}{dy}\right)^2\right]^{3/2}}{\frac{d^2x}{dy^2}}$$

Note:

1. The general form of eqn. of circle is $x^2+y^2+2gx+2fy+c=0$ where centre $(-g,-f)$ and radius $= \sqrt{g^2+f^2-c}$

2. The radius of curvature at any point on the circle = radius of the circle.
   curvature of the circle $= \frac{1}{r}$ where r is the radius of the circle.

3. curvature of the straight line is zero.

curvature

1]. Find the curvature at any pt. on the curve $x^2+y^2-6x-4y+10=0$

Soln.

The general form is $x^2+y^2+2gx+2fy+c=0$

Here $2g=-6 \Rightarrow g=-3$

$2f=-4 \Rightarrow f=-2$

$\text{centre} = (-g,-f) = (3,2)$

$\text{Radius} = \sqrt{g^2+f^2-c} = \sqrt{9+4-10} = \sqrt{3}$

$\rho = r = \sqrt{3}$

$\text{curvature} = \frac{1}{\rho} = \frac{1}{\sqrt{3}}$

2]. Find the curvature of $2x^2+2y^2+5x-2y+1=0$

Soln.

The general form is $x^2+y^2+2gx+2fy+c=0$

The given eqn. becomes,

$x^2+y^2+\frac{5}{2}x-y+\frac{1}{2}=0$

Here $2g=\frac{5}{2} \Rightarrow g=5/4$

$2f=-1 \Rightarrow f=-1/2$

$\text{centre} = (-g,-f) = (-5/4, 1/2)$

$\text{Radius} = \sqrt{g^2+f^2-c} = \sqrt{\frac{25}{16}+\frac{1}{4}-\frac{1}{2}} = \sqrt{\frac{21}{16}}$

$\rho = r = \frac{\sqrt{21}}{4}$

$\text{curvature} = \frac{1}{\rho} = \frac{4}{\sqrt{21}}$

3]. Find the curvature of $x^2+y^2=5$

Soln.

The general form is $x^2+y^2+2gx+2fy+c=0$

Here $2g = 0 \Rightarrow g = 0$

$2f = 0 \Rightarrow f = 0$

Centre $= (0, 0)$

Radius $\rho = \sqrt{5}$

curvature $= \dfrac{1}{\rho} = \dfrac{1}{\sqrt{5}}$

HW find the curvature of

1). $x^2 + y^2 + 4x - 6y - 1 = 0$

2). $3x^2 + 3y^2 + 9x + 18y - 5 = 0$

3). $2x^2 + 2y^2 = 3$