Period : 3

Problem :- 2

Reduce the matrix $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ the diagonal form by using orthogonal transformation.

Step:-1 The characteristic equation is

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$$

$$D_1 = 1+2+1 = 4$$

$$D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= (2-1) + (1-0) + (2-1)$$

$$= 1+1+1 = 3$$

$$D_3 = |A|$$

$$= 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} + 0$$

$$= 1(2-1) + 1(-1-0)$$

$$= 1-1 = 0$$

$$\therefore \lambda^3 - 4\lambda^2 + 3\lambda = 0$$

Eigen values :-

$$\lambda(\lambda^2 - 4\lambda + 3) = 0$$

$$\lambda = 0$$

And $\lambda^2 - 4\lambda + 3 = 0$

$$(\lambda - 3)(\lambda - 1) = 0$$

$$\lambda = 1, 3$$

$\therefore$ The eigen values are $0, 1, 3$

Eigen vectors :-

$$(A - \lambda I)x = 0$$

UNIT 1- EIGEN VALUE PROBLEMS **DIAGONALISATION OF THE MATRIX**

$$\begin{pmatrix}1-\lambda & -1 & 0\\ -1 & 2-\lambda & 1\\ 0 & 1 & 1-\lambda\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

Case:- 1 where $\lambda = 0$

$$\begin{pmatrix}1 & -1 & 0\\ -1 & 2 & 1\\ 0 & 1 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\begin{array}{cccc} x_1 & & x_2 & x_3 \\ -1 & 0 & 1 & -1 \\ 2 & 1 & -1 & 2 \end{array}$$

$$\frac{x_1}{(-1-0)} = \frac{x_2}{(0-1)} = \frac{x_3}{(2-1)}$$

$$\frac{x_1}{-1} = \frac{x_2}{-1} = \frac{x_3}{1}$$

$$X_1 = \begin{pmatrix}-1\\-1\\1\end{pmatrix}$$

Case:- 2 where $\lambda = 1$

$$\begin{pmatrix}0 & -1 & 0\\ -1 & 1 & 1\\ 0 & 1 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\begin{array}{cccc} x_1 & & x_2 & x_3 \\ -1 & 0 & 0 & -1 \\ 1 & 1 & -1 & 1 \end{array}$$

$$\frac{x_1}{(-1-0)} = \frac{x_2}{(0-0)} = \frac{x_3}{(0-1)}$$

$$\frac{x_1}{-1} = \frac{x_2}{0} = \frac{x_3}{-1}$$

$$X_2 = \begin{pmatrix}-1\\0\\-1\end{pmatrix}$$

case :- 3 where , $\lambda = 3$

$$\begin{pmatrix} -2 & -1 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{cccccc} & x_1 & & x_2 & & x_3 \\ -1 & & 0 & & -2 & & -1 \\ -1 & & 1 & & -1 & & -1 \end{array}$$

$$\frac{x_1}{(-1+0)} = \frac{x_2}{(0+2)} = \frac{x_3}{(2-1)}$$

$$\frac{x_1}{-1} = \frac{x_2}{2} = \frac{x_3}{1}$$

$$\therefore X_3 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$\Rightarrow$ Step :- 2

Now, $X_1^T \cdot X_2 = (-1 \ -1 \ 1)\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$

$$1 + 0 - 1 = 0$$

$$X_2^T \cdot X_3 = (-1 \ 0 \ -1)\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$$= 1 + 0 - 1 = 0$$

$$X_3^T \cdot X_1 = (-1 \ 2 \ 1)\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$= 1 - 2 + 1 = 0$$

$\therefore$ Eigen vectors are pair wise orthogonal.

$\Rightarrow$ step :- 3 Normalized Eigen vectors

$X_1 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ $\quad l(x) = \sqrt{(-1)^2 + (-1)^2 + (1)^2}$

$$= \sqrt{1+1+1} = \sqrt{3}$$

$$\therefore X_1 = \begin{pmatrix} -1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$$

$$x_2 = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \qquad l(x) = \sqrt{(-1)^2+(0)^2+(-1)^2}$$

$$= \sqrt{2}$$

$$x_2 = \begin{pmatrix} -1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix}$$

$$x_3 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \qquad l(x) = \sqrt{(-1)^2+(2)^2+(1)^2}$$

$$= \sqrt{1+4+1} = \sqrt{6}$$

$$x_3 = \begin{pmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{pmatrix}$$

$\Rightarrow$ Step : 4

Normalized model matrix

$$N = \begin{pmatrix} -1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ -1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6} \end{pmatrix}$$

$\Rightarrow$ Step :- 5

To check :- N is should be orthogonal

i.e, $NN^T = N^TN = I$

Now,

$$NN^T = \begin{pmatrix} -1/\sqrt{3} & -1/\sqrt{2} & -1/\sqrt{6} \\ -1/\sqrt{3} & 0 & 2/\sqrt{6} \\ -1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{pmatrix} \begin{pmatrix} -1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ -1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

N is orthogonal.

Step :- 6 $\quad D = N^T A N$

$$= \begin{pmatrix} -1/\sqrt{3} & -1/\sqrt{3} & -1/\sqrt{3} \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ -1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6} \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1/\sqrt{3} & -1/\sqrt{2} & -1/\sqrt{6} \\ -1/\sqrt{3} & 0 & 2/\sqrt{6} \\ -1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$