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Q. Find the eigen values and eigen vectors of the matrix:

i) $\begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$

Let $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$

Step 1:-

The characteristic equation is

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$$

$$D_1 = 2+1+1 = 4$$

$$D_2 = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (1-4) + (2-1) + (2-1)$$

$$= -3+1+1 = -2+1 = -1$$

$$D_3 = 2\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix}$$

$$= 2(1-4) - 1(1-2) - 1(-2+1)$$

$$= 2(-3) - 1(-1) - 1(-1)$$

$$= -6+1+1 = -5+1 = -4$$

$$\therefore \lambda^3 - 4\lambda^2 - \lambda + 4 = 0$$

Step:2 To find the eigen values

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -1 & 4 \\ & 0 & 1 & -3 & -4 \\ \hline & 1 & -3 & -4 & \boxed{0} \end{array}$$

$\lambda = 1$

$$\lambda^2 - 3\lambda - 4 = 0$$

$(\lambda - 4)(\lambda + 1) = 0$

$\lambda = 4, -1$

-4

$-4 \quad 1$

$= -3$

Step:- $\therefore$ Eigen values are $-1, 1, 4$

Step:- 3 To find the eigen vectors

$$(A - \lambda I)X = 0$$

$$\begin{pmatrix} 2-\lambda & 1 & -1 \\ 1 & 1-\lambda & -2 \\ -1 & -2 & 1-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Case :- 1

$\lambda = -1$

$$\begin{pmatrix} 3 & 1 & -1 \\ 1 & 2 & -2 \\ -1 & -2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By cross multiplication Rule

$$\begin{array}{cccc} x_1 & & x_2 & x_3 \\ 1 & -1 & 3 & 1 \\ 2 & -2 & 1 & 2 \end{array}$$

$$\frac{x_1}{(-2+2)} = \frac{x_2}{(-1+6)} = \frac{x_3}{(6-1)}$$

$$\frac{x_1}{0} = \frac{x_2}{5} = \frac{x_3}{5} \Rightarrow \frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$X_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Case :- 2

$\lambda = 1$

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 0 & -2 \\ -1 & -2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By cross multiplication rule,

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ 1 & -1 & 1 & 1 \\ 0 & -2 & 1 & 0 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 1 & -1 \\ 0 & -2 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} -1 & 1 \\ -2 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}}$$

$$\frac{x_1}{(-2+0)} = \frac{x_2}{(-1+2)} = \frac{x_3}{(0-1)} \Rightarrow \frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{-1}$$

$$X_2 = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$$

Case:- 3

$\lambda = 4$

$$\begin{pmatrix} -2 & 1 & -1 \\ 1 & -3 & -2 \\ -1 & -2 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By cross multiplication rule,

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ 1 & -1 & -2 & 1 \\ -3 & -2 & 1 & -3 \end{array}$$

$$\frac{x_1}{(-2-3)} = \frac{x_2}{(-1-4)} = \frac{x_3}{(+6-1)}$$

$$\frac{x_1}{-5} = \frac{x_2}{-5} = \frac{x_3}{5}$$

$$\frac{x_1}{-1} = \frac{x_2}{-1} = \frac{x_3}{1}$$

$$X_3 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

Conclusion:-

Eigen values	Eigen vectors
-1	$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$
1	$\begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$
4	$\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$