Period: 6

Eigen Values and Eigen Vectors

Solve the characteristic equation, we get eigen.

If there exist a non-zero vectors x such that $(A-\lambda I)\bar{x}=0$, then the vector x is called eigen vector.

Q1) Find the eigen values and eigen vectors of the matrix.

i) $\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$

Step 1 :- characteristic equation

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0 \longrightarrow (1)$$

D_1 = Sum of the main diagonal element

$= 7+6+5 = 18$

D_2 = Sum of the minors of the main diagonal elements.

$$= \begin{vmatrix} 6 & -2 \\ -2 & 5 \end{vmatrix} + \begin{vmatrix} 7 & 0 \\ 0 & 5 \end{vmatrix} + \begin{vmatrix} 7 & -2 \\ -2 & 6 \end{vmatrix}$$

$$= (30-4) + (35) + (42-4)$$

$$= 26 + 35 + 38 = 99$$

$$D_3 = |A|$$

$$= \begin{vmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{vmatrix}$$

$$7\begin{vmatrix} 6 & -2 \\ -2 & 5 \end{vmatrix} - (-2)\begin{vmatrix} -2 & -2 \\ 0 & 5 \end{vmatrix} + 0$$

$$7(30-4) + 2(-10)$$

$7(26) + 2(-10)$

$= 182 - 20 = 162$

$\therefore \lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0$

Step :- 2 To find eigen values

$$\begin{array}{c|cccc} 3 & 1 & -18 & 99 & -162 \\ & 0 & 3 & -45 & 162 \\ \hline & 1 & -15 & 54 & 0 \end{array}$$

$\lambda = 3.$

$\lambda^2 - 15 + 54 = 0$

$(\lambda - 6)(\lambda - 9) = 0$

$\lambda = 6, 9$

The eigen values are $3, 6, 9$.

Step 3 :- To find eigen vectors

$(A - \lambda I)X = 0$

$$\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

case 1, when $\lambda = 3$

$$\begin{pmatrix} 4 & -2 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By cross multiplication rule,

$$\begin{array}{cccc} x_1 & & x_2 & x_3 \\ -2 & 0 & 4 & -2 \\ 3 & -2 & -2 & 3 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} -2 & 0 \\ 3 & -2 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0 & 4 \\ -2 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & -2 \\ -2 & 3 \end{vmatrix}}$$

$$\frac{x_1}{4-0} = \frac{x_2}{0+8} = \frac{x_3}{12-4}$$

$$\frac{x_1}{4} = \frac{x_2}{8} + \frac{x_3}{8} \Rightarrow \frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}$$

$$X_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

Case:- 2 when $\lambda = 6$

$$\begin{pmatrix} 1 & -2 & 0 \\ -2 & 0 & -2 \\ 0 & -2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By cross multiplication method

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -2 & 0 & 1 & -2 \\ 0 & -2 & -2 & 0 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0 & 1 \\ -2 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & -2 \\ -2 & 0 \end{vmatrix}}$$

$$\frac{x_1}{(4-0)} = \frac{x_2}{(0+2)} = \frac{x_3}{(0-4)}$$

$$\frac{x_1}{4} = \frac{x_2}{2} = \frac{x_3}{-4}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

$$X_2 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

case :- 3 $\lambda = 9$

$$\begin{pmatrix} -2 & -2 & 0 \\ -2 & -3 & -2 \\ 0 & -2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By cross multiplication rule,

$$\begin{array}{cccc} x_1 & & x_2 & & x_3 \\ -2 & 0 & -2 & -2 \\ -3 & -2 & -2 & -3 \end{array}$$

$$\begin{array}{cccccc} \multicolumn{2}{c}{x_1} & \multicolumn{2}{c}{x_2} & \multicolumn{2}{c}{x_3} \\ -2 & 0 & 0 & -2 & -2 & -2 \\ -3 & -2 & -2 & -2 & -2 & -3 \end{array}$$

$$\frac{x_1}{(4+0)} = \frac{x_2}{(0-4)} = \frac{x_3}{(6-4)}$$

$$\frac{x_1}{4} = \frac{x_2}{-4} = \frac{x_3}{2}$$

$$\frac{x_1}{2} = \frac{x_2}{-2} = \frac{x_3}{1}$$

$$X_3 = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

Conclusion:-

Eigen values	Eigen vectors
3	$\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$
6	$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$
9	$\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

Period: 3

Find the eigen values & eigen vectors of matrix.

9, $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$

let $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$

Step 1:- The characteristic equation

$D_1 = 1 + 2 + 3$

$D_1 = 6$

$D_2 = \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$

$= (6-2) + (3+2) + (2-0)$

$= 4 + 5 + 2 = 11$

$D_3 = 1 \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} - 0 + (-1) \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix}$

$= 1(6-2) - 0 - 1(2-4)$

$= 1(4) - 0 - 1(-2) = 4 - 0 - 1(-2) = 6$

$\Rightarrow 4+2 = 6 \quad \therefore \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

Step :- 2

To find eigen values

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & 0 & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$\lambda = 1$

$\lambda^2 - 5\lambda + 6 = 0$

$(\lambda - 2)(\lambda - 3) = 0$

$\lambda = 1, 2, 3$

Step :- 3 To find eigen vector

$(A - \lambda I)x = 0$

$$\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow \lambda = 1$

$$\begin{pmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By cross multiplication rule,

Case :- 1

$\Rightarrow$ $\lambda = 1$

$$\begin{array}{cccc} x_1 & x_2 & & x_3 \\ & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & & & \end{array}$$

$$\frac{x_1}{\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}} = \frac{x_2}{\begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}} = \frac{x_3}{\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}}$$

$$= \frac{x_1}{(0+1)} = \frac{x_2}{(-1-0)} = \frac{x_3}{(0)}$$

$$= \frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{0}$$

$$X_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

Case :- 2

$\Rightarrow$ $\lambda = 2$

$$\begin{pmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

By cross multiplication rule

$$\begin{array}{ccccc} x_1 & & x_2 & & x_3 \\ 0 & 1 & 1 & & 0 \\ 2 & 1 & 2 & & 2 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix}}$$

$$\frac{x_1}{(0-2)} = \frac{x_2}{(2-1)} = \frac{x_3}{(2-0)}$$

$$= \frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{2}$$

$$= \frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{2}$$

$$X_2 = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$$

Case :- 3

$\lambda = 3$

$$\begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix}$$

x_1	x_2		x_3
0	-1	-2	0
-1	1	1	-1

$$= \frac{x_1}{\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} -1 & -2 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix}}$$

$$= \frac{x_1}{(0-1)} = \frac{x_2}{(-1+2)} = \frac{x_3}{(2-0)}$$

$$= \frac{x_1}{-1} = \frac{x_2}{1} = \frac{x_3}{2}$$

$$X_3 = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

Conclusion :-

Eigen values	Eigen vectors
1	$\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$
2	$\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$
3	$\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

Q) Find the eigen values and eigen vectors of the matrix.

3) $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$ 4) $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

Q// 3) Find the eigen values and eigen vectors of matrix

$$\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$$

Step:-1

$D_1 = 2+1-3 = 0$

$D_2 = (-3-2)+(-6+0)+(2-4)$

$= -5-6-2$

$= -11-2 = -13$

$D_3 = 2(-3-2)-(2)(-6+7)+0$

$= 2(-5)-2(1) = -10-2 = -12$

$\lambda^3 - 0\lambda^2 + (-13)\lambda + 12 = 0$

$\therefore \lambda^3 - 13\lambda + 12 = 0$

Step 2:-

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -13 & 12 \\ & 0 & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$\lambda = 1$

$\lambda^2 + \lambda - 12 = 0$

$(\lambda+4)(\lambda-3) = 0$

$\lambda = 3, -4$

$\therefore$ Eigen values $\lambda = 1, 3, -4$

Step 3:- To find the Eigen vectors

$(A-\lambda I)X = 0$

$$\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2-\lambda & 2 & 0 \\ 2 & 1-\lambda & 1 \\ -7 & 2 & -3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

case :- 1

$\lambda = 1$

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ -7 & 2 & -4 \end{pmatrix}$$

$$\begin{matrix} x_1 & & x_2 & & x_3 & \\ 2 & 0 & 1 & 2 \\ 0 & 1 & 2 & 0 \end{matrix}$$

$$\frac{x_1}{\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}} = \frac{x_2}{\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}} = \frac{x_3}{\begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}}$$

$$\frac{x_1}{(2-0)} = \frac{x_2}{(0-1)} = \frac{x_3}{(0-4)}$$

$$\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{-4}$$

$$\therefore X_1 = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$$

case :- 2

$\lambda = 3$

$$\begin{pmatrix} -1 & 2 & 0 \\ 2 & -2 & 1 \\ -7 & 2 & -6 \end{pmatrix}$$

$$\begin{matrix} x_1 & & x_2 & & x_3 & \\ 2 & 0 & -1 & 2 \\ -2 & 1 & 2 & -2 \end{matrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 0 \\ -2 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 2 \\ 2 & -2 \end{vmatrix}}$$

$$\frac{x_1}{(2+0)} = \frac{x_2}{(0+1)} = \frac{x_3}{(2-4)}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

$$X_2 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

case 3 :-

$\lambda = -4$

$$\begin{pmatrix} 2-\lambda & 2 & 0 \\ 2 & 1-\lambda & 1 \\ -7 & 2 & -3-\lambda \end{pmatrix} = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 5 & 1 \\ -7 & 2 & 1 \end{pmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0 & 6 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 6 & 2 \\ 2 & 5 \end{vmatrix}}$$

$$\frac{x_1}{(2-0)} = \frac{x_2}{(0-6)} = \frac{x_3}{(30-4)}$$

$$\frac{x_1}{2} = \frac{x_2}{-6} = \frac{x_3}{26}$$

$$\frac{x_1}{1} = \frac{x_2}{-3} = \frac{x_3}{13}$$

$$X_3 = \begin{pmatrix} 1 \\ -3 \\ 13 \end{pmatrix}$$

conclusion:-

Eigen values	eigen vectors
1	$\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$
3	$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$
-4	$\begin{pmatrix} 1 \\ -3 \\ 13 \end{pmatrix}$

Q_{11} Find eigen values and eigen vectors of matrix.

A) $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

Step:-1 $D_1 = 18 + 7 + 3 = 18$

$$D_2 = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$$

$$= (2 - 16) + (24 - 4) + (56 - 36)$$

$$= 5 + 20 + 20 = 45$$

$$D_3 = 8\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} - (-6)\begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} -6 & 7 \\ 2 & -4 \end{vmatrix}$$

$$= 8(21 - 16) + 6(-18 + 8) + 2(24 - 14)$$

$$= 40 - 60 + 20 = 0$$

$$\therefore \lambda^3 - 18\lambda^2 + 45\lambda - 0 = 0$$

Step: 2 To find eigen values

$$\begin{array}{c|cccc} 3 & 1 & -18 & 45 & 0 \\ & 0 & 3 & -45 & 0 \\ \hline & 1 & -15 & 0 & \boxed{0} \end{array}$$

$\lambda^2 - 15\lambda + 0 = 0$

$\lambda(\lambda - 15) = 0$

$$\lambda^3 - 18\lambda^2 + 45\lambda - 0 = 0$$

$$\lambda(\lambda^2 - 18\lambda + 45) = 0$$

$$\boxed{\lambda = 0}$$

45

$-15 \quad -3$

$$\lambda^2 - 15\lambda - 3\lambda + 45 = 0$$

$$\lambda(\lambda - 15) - 3(\lambda - 15) = 0$$

$$(\lambda - 15)(\lambda - 3) = 0$$

$$\lambda = 15, 3$$

$\therefore$ Eigen values are $0, 3, 15$

Step:3 The Eigen vectors

$$(A-\lambda I)X=0$$

$$\left(\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

Case :- 1

$\lambda = 0$

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ 7 & -4 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 2 & 8 \\ -4 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}}$$

$$\frac{x_1}{(24-14)} = \frac{x_2}{(-12+32)} = \frac{x_3}{(56-36)}$$

$$\frac{x_1}{10} = \frac{x_2}{20} = \frac{x_3}{20}$$

$$X_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

Case :- 2

$\lambda = 3$

$$\begin{pmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ 4 & -4 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 2 & 5 \\ -4 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -6 \\ -6 & 4 \end{vmatrix}}$$

$$\frac{x_1}{(24-8)} = \frac{x_2}{(-12+20} = \frac{x_3}{(120-36)}$$

$$\frac{x_1}{16} = \frac{x_2}{8} = \frac{x_3}{-16}$$

$$X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

Case :- 3

$\lambda = 15$

$$\begin{vmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{vmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ -8 & -4 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 2 & -7 \\ -4 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & -6 \\ -6 & -8 \end{vmatrix}}$$

$$\frac{x_1}{(24+16)} = \frac{x_2}{(-12-28)} = \frac{x_3}{(56-36)}$$

$$\frac{x_1}{40} = \frac{x_2}{-40} = \frac{x_3}{20}$$

$$X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

conclusion :-

Eigen values	Eigen vectors
0	$\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$
3	$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$
15	$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$