DEFENITION OF QUADRATIC FORM:

A homogeneous polynomial of degree 2 with any number of variables are known as quadratic form

$$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 0$$

$$\begin{bmatrix} co\ eff\ of\ x^2 & \frac{1}{2} co\ eff\ of\ xy & \frac{1}{2} co\ eff\ of\ xz \\ \frac{1}{2} co\ eff\ of\ yx & co\ eff\ of\ x^2 & \frac{1}{2} co\ eff\ of\ yz \\ \frac{1}{2} co\ eff\ of\ zx & \frac{1}{2} co\ eff\ of\ zy & co\ eff\ of\ z^2 \end{bmatrix}$$

WORKING RULE:

STEP 1:Write the matrix o the quadratic form .then find D=$N^T AN$

By orthogonal transformation.

STEP 2: Find $Q = Y^T DY$

INDEX OF QUADRATIC FORM

The no of positive square terms in the canonical form is called the index of the quadratic form.It is denoted by p

SIGNATURE OF QUADRATIC FORM

The different of positive and negative square terms are called signature of quadratic terms .denoted by s

s=2p-r

NATURE OF QUDARTIC FORM :

Positive definite Ex: 1,2,2

Negative definite Ex: -1,-2-,2

Semi Positive Ex: 0,1,2

Semi negative Ex: 0,-1,-2

Indefinite Ex: -1,1,2

-2,-1,1

Problems:

1.Find the nature of the given equation $2x^2 + 2xy + 3y^2 = 0$

STEP 1:The matrix form

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$$

c_1 = Sum Of Diagonal Elements

= 2+3 =5

c_2 = $|A|$ =$\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}$ =5

The characteristic equation is

$\lambda^2 - 5\lambda + 5 = 0$

Here $c_1 and\ c_2$ are positive

Hence the nature of the given matrix is positive definite