Find e.values & e.vectors of $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

Given $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$

Step 1 :- To find char. eqn

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

where $S_1 = 7$; $S_2 = 14$; $S_3 = 8$

$\therefore$ The char. eqn. is $\lambda^3 - 7\lambda^2 + 14\lambda - 8 = 0$.

Step 2 :- To find e.values.

$$\lambda^3 - 7\lambda^2 + 14\lambda - 8 = 0$$

$$\lambda = 1, 2, 4.$$

Step 3: To find e.vector

$$(A - \lambda I)x = 0$$

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right] \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 3-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Case (i) : when $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since I row is zero th row, by taking cofactor for I row we get.

$$\frac{x_1}{\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & -1 \\ 0 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 2 \\ 0 & -1 \end{vmatrix}}$$

$$\frac{x_1}{3} = \frac{-x_2}{0} = \frac{x_3}{0}$$

$\therefore$ E. vector $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$

Case (ii): when $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since ② & ③ row are same, by taking cofactor for II row we get

$$\frac{x_1}{\begin{vmatrix} 0 & 0 \\ 0 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}}$$

$$\frac{x_1}{0} = \frac{-x_2}{1} = \frac{x_3}{-1}$$

$\therefore$ E. vector $X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$

Case (iii): when $\lambda = 4$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since ② & ③ row are same, by taking cofactor for Ist row we get

$$\frac{x_1}{0} = -\frac{x_2}{3} = \frac{x_3}{3}$$

$$\therefore \text{E. vector } X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix}$$

Find E. values & E. vectors of $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{pmatrix}$

Let $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{pmatrix}$

Step 1: To find char. eqn. $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = -1$; $S_2 = -21$; $S_3 = 45$

$\therefore$ The char. Eqn. is $\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

Step 2: To find E. values.

$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

$\lambda = -3,\ \lambda = -3,\ \lambda = 5$

E. values are $-3, -3, 5$

Step 3: To find E. vector. $(A-\lambda I)x = 0$

$$\left[\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Case (i): when $\lambda = -3$

$$\begin{pmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & 2 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

since all the three rows are same, conside any one of the row as eqn.

$$x_1 + 2x_2 - 3x_3 = 0$$

put $x_1 = 0 \Rightarrow 2x_2 - 3x_3 = 0$

$$\frac{x_2}{3} = \frac{x_3}{2}$$

$\therefore$ E. vector $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$

put $x_2 = 0 \Rightarrow x_1 - 3x_3 = 0$

$$x_1 = 3x_3$$

$$\frac{x_1}{3} = \frac{x_3}{1}$$

$\therefore$ E. vector $X_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

Case (ii): when $\lambda = 5$

$$\begin{pmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Since all the three rows are diff. by taking cofac for 1 rows we get

$$\frac{x_1}{\begin{vmatrix} -4 & -6 \\ -2 & -5 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 2 & -6 \\ -1 & -5 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 2 & -4 \\ -1 & -2 \end{vmatrix}}$$

$$\frac{x_1}{8} = \frac{-x_2}{-16} = \frac{x_3}{-8}$$

$\therefore$ E. vector $X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ +16 \\ -8 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$