Properties :-

1) If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the E.values of matrix A, then $k\lambda_1, k\lambda_2, \ldots k\lambda_n$ are the E.values of matrix kA, where k is non-zero scalar.

2) If $\lambda_1, \lambda_2, \ldots \lambda_n$ are the E.values of matrix A, then $\lambda_1^p, \lambda_2^p, \ldots \lambda_n^p$ are the E.values of A^p, where p is the +ve integer.

3) If $\lambda_1, \lambda_2, \ldots \lambda_n$ are the E.values of A, then $1/\lambda_1, 1/\lambda_2, \ldots 1/\lambda_n$ are the E.values of A^{-1}, provided $\lambda_r \neq 0$ where $r = 1, \ldots n$.

4) E.values of a real symmetric are all real

5) The square matrix and its transpose A^T have the same E values

6) A sum of the E.values of the matrix A is equal to sum of the main diagonal elt (ie) Trace of A

7) The product of E.values of the matrix A is equal to $|A|$

8) If $\lambda_1, \lambda_2, \ldots \lambda_n$ are the E values of A, Then $\lambda_1 - k, \lambda_2 - k, \ldots \lambda_n - k$ are the E values of $A - kI$

9) If λ is the e.value of A, then $\frac{|A|}{\lambda}$ is the e.value of adj A

10) E.values of a diagonal matrix are the diagonal elts.

11) E.values of an upper & lower triangular matrix are the diagonal elts.

problems based on properties:

1) Find the sum & product of all e.values of matrix.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & 3 \end{bmatrix}$$

Sum of e.values = Sum of main diagonal elts (by prop. 6)

$= 1+0+3 = 4$

product of e.values $= |A|$

$= 1(3) - 2(3+6) - 2(-1) = -13$

2) Find the e.values of $A^2, A^{-1}, 3A$, & $A-5I$ when

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

Soln: A is an upper triangular matrix.

E. values of A are 3, 2, 5 (by prop 1)
E. values of A^2 are $3^2, 2^2, 5^2$ (or) 9, 4, 25 (by prop 2)
E. values of A^{-1} are $1/3, 1/2, 1/5$ (by prop. 3)
E. values of 3A are $3\times3, 3\times2, 3\times5$ (or) 9, 6, 15 (by prop. 1)
E. values of $A-5I$ are $3-5, 2-5, 5-5$ (or) $-2, -3, 0$ (by prop. 8)

3) If 3 & 15 are the two E. values of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ find $|A|$ without expanding the determinant.

Soln: The E. values are 3 & 15

Let λ be the 3rd E. value.

Sum of E. value = sum of main diagonal elt.

$3+15+\lambda = 8+7+3.$

$18+\lambda = 18$

$\lambda = 0$

∴ By prop 7, product of E. value = $|A|$

$\Rightarrow 3\times15\times0 = |A|$

$0 = |A|.$

4) Find the E.values of adj A, if $A = \begin{bmatrix} 8 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

Soln: A is an upper triangular matrix.

$\Rightarrow$ E. values of A are 8, 4, 1.

$|A|$ = product of E. values.

$= 8 \times 4 \times 1$

$= 32$.

∴ The E. values of adj A are $\frac{32}{8}, \frac{32}{4}, \frac{32}{1}$

(i.e) 4, 8, 32.

5) Two E.values of $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ are equal & they are double the third. Find the E.value of A^2

Soln: Let λ be the third E. value then 2 E.values are $2\lambda, 2\lambda$.

Sum of E.values = Sum of main diagonal elts.

$\lambda + 2\lambda + 2\lambda = 4 + 3 - 2$

$5\lambda = 5$

$\lambda = 1$

∴ E. values of A are 2, 2, 1

E values of A^2 are 4, 4, 1 (by prop. 2)

6) If 3 & 6 are the E.values of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ write down the E.values of A^{-1} & $3A$.

Sol: 3 & 6 are two E.values.

Sum of E.values = Sum of main diagonal elts

$3+6+\lambda = 1+5+1$

$9+\lambda = 7$

$\lambda = -2$

∴ E values are $-2, 3, 6$.

E values of A^{-1} are $-1/2, 1/3, 1/6$

E. values of $3A$ are $-6, 9, 18$

7) The pdt of two E.values of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16 Find third E.value.

Let λ be the third E. value

product of E.value = $|A|$

$16\lambda = 6(9-1) + 2(-6+2) + 2(2-6)$

$= 32$

$\lambda = 2$, which is third E.value

8) Find the E.value of $\begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$ corresponding to the E.vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

WKT $(A - \lambda I)x = 0$

$$\left[\begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 3 \\ 0 & 4-\lambda \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2 - \lambda + 0 = 0$$

$\lambda = 2$, The corresponding E.value