Fourier Sine Transform :

The infinite Fourier Sine Transform of $f(x)$ is defined by $F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin(sx)\, dx = F_s(s)$

The Inverse Fourier Sine Transform of $f(x)$ is defined by $F_s^{-1}[F_s[f(x)]] = \sqrt{\frac{2}{\pi}} \int_0^\infty F_s[f(x)] \sin(sx)\, ds$

$F_s[f(x)]$ and $F_s^{-1}[F_s[f(x)]]$ are called Fourier Sine Transform Pair.

3) Find the fourier Sine Transform of e^{-ax}, $a>0$

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin(sx)\, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \sin(sx)\, dx$$

$$= \sqrt{\frac{2}{\pi}} \left(\frac{s}{s^2+a^2} \right)$$

4) Find the fourier Sine Transform of $f(x) = \frac{1}{x}$, $x>0$

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin(sx)\, dx$$

formula: $\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{1}{x} \sin(sx)\, dx = \sqrt{\frac{2}{\pi}} \times \frac{\pi}{2} = \sqrt{\frac{\pi}{2}}$$

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$$F_c\left[5e^{-2x}+2e^{-5x}\right] = \sqrt{\frac{2}{\pi}}\int_0^\infty (5e^{-2x}+2e^{-5x})\cos sx\,dx$$

$$= \sqrt{\frac{2}{\pi}}\left\{5\int_0^\infty e^{-2x}\cos(sx)\,dx + 2\int_0^\infty e^{-5x}\cos(sx)\,dx\right\}$$

$$= \sqrt{\frac{2}{\pi}}\left\{5\,\frac{2}{s^2+4} + 2\cdot\frac{5}{s^2+25}\right\}$$

$$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$$

$$= 10\sqrt{\frac{2}{\pi}}\left[\frac{1}{s^2+4} + \frac{1}{s^2+25}\right]$$

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b) Find the Fourier Sine Transform of $3e^{-5x}+5e^{-2x}$

Soln:

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin(sx)\,dx$$

$$F_s\left[3e^{-5x}+5e^{-2x}\right] = \sqrt{\frac{2}{\pi}}\int_0^\infty (3e^{-5x}+5e^{-2x})\sin sx\,dx$$

$$= \sqrt{\frac{2}{\pi}}\left[\int_0^\infty 3e^{-5x}\sin(sx)\,dx + \int_0^\infty 5e^{-2x}\sin(sx)\,dx\right]$$

$$= \sqrt{\frac{2}{\pi}}\left[3\left(\frac{s}{s^2+25}\right) + 5\left(\frac{s}{s^2+4}\right)\right]$$

formula:

$$\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2}$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{3s}{s^2+25} + \frac{5s}{s^2+4}\right]$$ //

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3) Find the Fourier Sine Transform of $f(x) = \begin{cases} x & 0<x<1 \\ 2-x & 1<x<2 \\ 0 & x>2 \end{cases}$

Soln: $$F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin(sx)\,dx.$$

$$= \sqrt{\frac{2}{\pi}}\left\{\int_0^1 (x)\sin(sx)\,dx + \int_1^2 (2-x)\sin(sx)\,dx + 0\right\}$$

$$= \sqrt{\frac{2}{\pi}}\left[\left(x\left(\frac{-1}{s}\cos(sx)\right) + \frac{1}{s^2}\sin(sx)\right)_0^1 + \left[(2-x)\left(\frac{-1}{s}\cos(sx)\right) + \left(\frac{-1}{s^2}\right)\sin(sx)\right]_1^2\right]$$

$$= \sqrt{\frac{2}{\pi}} \left[\left(-\frac{\cos s}{s} + \frac{\sin s}{s^2} - 0 + 0 \right) + \left(0 - \frac{\sin 2s}{s^2} + \frac{\cos s}{s} + \frac{\sin s}{s^2} \right) \right]$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{2 \sin s}{s^2} - \frac{\sin 2s}{s^2} \right] = \sqrt{\frac{2}{\pi}} \frac{1}{s^2} \left[2 \sin s - \sin 2s \right]$$

$$= \sqrt{\frac{2}{\pi}} \frac{1}{s^2} \left[2 \sin s - 2 \sin s \cos s \right]$$

$$= \sqrt{\frac{2}{\pi}} \frac{2 \sin s}{s^2} \left[1 - \cos s \right]$$

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}} \frac{2 \sin s}{s^2} (1 - \cos s)$$

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