PROPERTIES OF EIGEN VALUES

1. The sum of eigen values of the matrix A is equal to the sum of its diagonal elements. i,e, Trace of A
2. The product of the eigen values of a matrix A is equal to its determinant.
3. If λ_1,λ_2,…λ_n are the eigen values of matrix A then $\frac{1}{\lambda_1}$,$\frac{1}{\lambda_2}$,…..$\frac{1}{\lambda_n}$are the eigen values of A^{-1}.
4. If λ_1,λ_2,…λ_n are the eigen values of matrix A then λ_1^m,λ_2^m,….λ_n^m are the eigen values of A^m.(m is a positive integer)
5. If λ_1,λ_2,…λ_n are the eigen values of matrix A then $k\lambda_1$,$k\lambda_2$,…. $k\lambda_n$ are the eigen values of the matrix kA
6. If λ_1,λ_2,…λ_n are the eigen values of matrix A then the matrix $A + kI$ has the eigen values $k + \lambda_1$, $k + \lambda_2$,…. $k + \lambda_n$
7. If λ_1,λ_2,…λ_n are the eigen values of matrix A then the matrix $A - kI$ has the eigen values $\lambda_1 - k$,$\lambda_2 - k$,….$\lambda_n - k$
8. Every square matrix and its transpose have the same eigen values.
9. The eigen values of a triangular matrix are its diagonal elements.

PROPERTIES OF EIGEN VECTORS

1. If all the eigenvalues λ_1,λ_2,…λ_n of a matrix are distinct then the corresponding eigen vectors x_1,x_2,…x_n will be linearly independent.
2. If two or more eigen values of a matrix are equal, then the eigen vectors may be either linearly independent or linearly dependent.
3. The eigen vectors corresponding to distinct eigen values of real symmetric matrix is orthogonal.

PROBLEMS BASED ON PROPERTIES

1. Find the sum and product of Eigen values of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

Solution:

Sum of Eigen values of a matrix A = Sum of main diagonal elements of its matrix

$$= 3 + 5 + 3$$

$$= 11$$

Product of Eigen values of a matrix A = |A|

$$= \begin{vmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{vmatrix}$$

$$= 3(15 - 1) + 1(-3 + 1) + 1(1 - 5)$$

$$= 42 - 2 - 4$$

$$= 36$$

2. The Product of Eigen values of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value.

Solution:

Let the eigen values be $\lambda_1, \lambda_2, \lambda_3$

Given

$$\lambda_1\lambda_2 = 16 \rightarrow (1)$$

We know that,

Product of Eigen values of a matrix A = |A|

$$\lambda_1\lambda_2\lambda_3 = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$= 6(9 - 1) + 2(-6 + 2) + 2(2 - 6)$$

$$= 48 - 8 - 8$$

$\lambda_1\lambda_2\lambda_3 = 32 \rightarrow (2)$

Sub (1) in (2) , we get $16\lambda_3 = 32$

$$\Lambda_3 = 2$$

Hence the third eigen value is 2.

3. If 2 and 3 are the eigen values of $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$. Find the eigen value of A^{-1}.

Solution:

Let the eigen values be $\lambda_1, \lambda_2, \lambda_3$

Given, two eigen values are 2 and 3.

$$\lambda_1 = 2\ ,\ \lambda_2 = 3 \rightarrow (1)$$

We know that, λ

Sum of Eigen values of a matrix A = Sum of main diagonal elements of its matrix

$$\lambda_1 + \lambda_2 + \lambda_3 = 3 - 3 + 7$$

$\lambda_1 + \lambda_2 + \lambda_3 = 7 \rightarrow (2)$

Sub (1) in (2) we get

$$2 + 3 + \lambda_3 = 7$$

$$\lambda_3 = 2$$

The eigen values of A^{-1} are ${\lambda_1}^{-1}, {\lambda_2}^{-1}, {\lambda_3}^{-1}$

i.e, $(2)^{-1}, (3)^{-1}, (2)^{-1}$

i.e, $\frac{1}{2}, \frac{1}{3}, \frac{1}{2}$