TYPE IV : RHS $= f(x,y) = e^{ax+by} x^m y^n$ (or) $e^{ax+by} \cos(ax+by)$ / $\sin(ax+by)$

$$P.I = \frac{1}{\phi(D,D')} e^{ax+by} \cdot x^m y^n .$$

Replace $D \to D+a$; $D' \to D'+b$. Then type III rule (or) type II rule .

1) Solve: $(D^2 - 2DD' + D'^2)z = x^2y^2 . e^{x+y}$

Soln: A.E. is $m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = +1, +1$

∴ the roots are real and equal .

C.F is $z = f_1(y+mx) + x f_2(y+mx)$

$= f_1(y+x) + x f_2(y+x)$

P.I : $P.I = \frac{1}{D^2 - 2DD' + D'^2} . x^2y^2 e^{x+y}$

Replace $D \to D+1$; $D' \to D'+1$

$$= \frac{1}{(D+1)^2 - 2(D+1)(D'+1) + (D'+1)^2} . e^{x+y} . x^2y^2$$

$$= \frac{1}{D^2+2D+1-2[DD'+D+D'+1]+D'^2+2D'+1} \cdot e^{x+y} \cdot x^2y^2$$

$$= \frac{1}{D^2+\cancel{2D}+\cancel{1}-2DD'-\cancel{2D}-\cancel{2D'}-\cancel{2}+D'^2+\cancel{2D'}+\cancel{1}} \cdot e^{x+y} \cdot x^2y^2$$

$$= \frac{1}{D^2-2DD'+D'^2} \, e^{x+y} \cdot x^2y^2$$

$$= e^{x+y} \frac{1}{D^2}\left[1-\left(\frac{2D'}{D}-\frac{D'^2}{D^2}\right)\right]^{-1} x^2y^2$$

$$= e^{x+y} \frac{1}{D^2}\left[1+\left(\frac{2D'}{D}-\frac{D'^2}{D^2}\right)\right] x^2y^2$$

$$= e^{x+y} \cdot \frac{1}{D^2}\left[x^2y^2+\frac{2D'}{D}x^2y^2-\frac{D'^2}{D^2}x^2y^2\right]$$

$$= e^{x+y}\left[\frac{1}{D^2}(x^2y^2)+\frac{2}{D^2}\left(2\frac{x^3}{3}y\right)-\frac{1}{D^2}\left(\cancel{2}\frac{x^4}{\cancel{12}_{6}}\right)\right]$$

$$= e^{x+y}\left[\frac{x^4}{12}y^2+\frac{4}{3}y\frac{x^5}{\cancel{20}_{5}}-\frac{1}{6}\frac{x^6}{30}\right]$$

$$= e^{x+y}\left[\frac{x^4y^2}{12}+\frac{x^5y}{15}-\frac{x^6}{180}\right]$$

$\therefore$ Solution is $z = C.F + P.I$

$$= f_1(y+x)+x f_2(y+x)+e^{x+y}\left[\frac{x^4y^2}{12}+\frac{x^5y}{15}-\frac{x^6}{180}\right]$$

Solve $r+s-6t = y\cos x$

Soln. :

Given $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$

$$(D^2 + DD' - 6D'^2) z = y\cos x$$

AE $\quad m^2+m-6=0$

$$(m+3)(m-2)=0$$

$$m = -3, 2$$

$$CF = f_1(y-3x) + f_2(y+2x)$$

$$PI = \frac{1}{(D^2+DD'-6D'^2)} y\cos x$$

Scanned by CamScanner

factor $\rightarrow D - 2D'$

where $y = c - 2x$

$D \rightarrow c$

$D' \rightarrow x$

$$= \frac{1}{(D+3D')(D-2D')} y \cos x$$

$$= \frac{1}{(D+3D')} \int (c-2x) \cos x \, dx$$

$$= \frac{1}{D+3D'} \left[(c-2x) \sin x - (-2)(-\cos x) \right]$$

$$= \frac{1}{D+3D'} \left[y \sin x - 2\cos x \right]$$

factor $\rightarrow D + 3D'$

$y \rightarrow c + 3x$

$$= \int \left[(c+3x) \sin x - 2 \cos x \right] dx$$

$$= (c+3x)(-\cos x) - 3(-\sin x) - 2 \sin x$$

$$= -y \cos x + 3 \sin x - 2 \sin x$$

$$= -y \cos x + \sin x$$

1) Solve: $(D^2 - 2DD')z = e^{2x} + x^3y$

Soln: A.E is $m^2 - 2m = 0$

$m(m-2) = 0$

$m_1 = 0, m_2 = 2$

$\Rightarrow$ the roots are real & different.

C.F is $z = f_1(y + m_1x) + f_2(y + m_2x)$

$= f_1(y) + f_2(y + 2x)$

P.I :

$$P.I_1 = \frac{1}{D^2 - 2DD'} \cdot e^{2x}$$

Replace $D \to 2, D' \to 0$

$$= \frac{1}{4-0} \cdot e^{2x}$$

$$= \frac{e^{2x}}{4}$$

$$P.I_2 = \frac{1}{D^2 - 2DD'} \cdot x^3y$$

$$= \frac{1}{D^2\left[1 - 2\frac{D'}{D}\right]} x^3y$$

$$= \frac{1}{D^2}\left[1 - 2\frac{D'}{D}\right]^{-1} x^3y$$

$$= \frac{1}{D^2}\left[1 + 2\frac{D'}{D} + \left(\frac{2D'}{D}\right)^2\right](x^3y)$$

$$= \frac{1}{D^2}\left[x^3y + \frac{2D'}{D}(x^3y) + 4\frac{D'^2}{D^2}(x^3y)\right]$$

$$= \frac{1}{D^2}\left[x^3y + \frac{2}{D}(x^3)\right]$$

$$= \frac{1}{D^2}\left[x^3y\right] + \frac{\cancel{2}}{D^2}\left[\frac{x^4}{\cancel{4}_2}\right]$$

$$= \frac{x^5}{20}y + \frac{x^6}{60}$$

$\therefore$ Solution is $z = C.F + P.I$

$$= f_1(y) + f_2(y + 2x) + \frac{e^{2x}}{4} + \frac{x^5}{20}y + \frac{x^6}{60}$$