Formation of difference equations:

Difference equations: A differential equation is a relation between the differences of an unknown function at one or more general values of the argument.

Thus $\Delta y_{(n+1)} + y_{(n)} = 2$

and $\Delta y_{(n+1)} + \Delta^2 y_{(n-1)} = 1$ are difference equations.

Order of a difference equation: The order of a difference equation is the difference between the largest and the smallest arguments occuring in the difference equation divided by the unit of increment.

(1) Form the difference equation corresponding to the family of curves $y = ax + bx^2$.

Soln:

$$y_x = ax + bx^2 \longrightarrow (1)$$

$$y_{x+1} = a(x+1) + b(x+1)^2 \longrightarrow (2)$$

$$y_{x+2} = a(x+2) + b(x+2)^2 \longrightarrow (3)$$

Eliminating a and b from (1), (2) & (3), we get

$$\begin{vmatrix} y_x & x & x^2 \\ y_{x+1} & x+1 & (x+1)^2 \\ y_{x+2} & x+2 & (x+2)^2 \end{vmatrix} = 0$$

$$y_x\left[(x+1)(x+2)^2 - (x+2)(x+1)^2\right] - y_{x+1}\left[x(x+2)^2 - x^2(x+2)\right] + y_{x+2}\left[x(x+1)^2 - x^2(x+1)\right] = 0$$

$$y_x(x+1)(x+2) - y_{x+1}\,2x(x+2) + y_{x+2}\,x(x+1) = 0$$

$$(x^2+3x+2)\,y_x - 2(x^2+2x)\,y_{x+1} + (x^2+x)\,y_{x+2} = 0$$

(19)

(2) ~~Form~~ From $y_n = a\cdot 2^n + b(-2)^n$, derive a difference eqn by eliminating the constants.

Soln:

$$y_n = a.2^n + b(-2)^n \longrightarrow (1)$$

$$y_{n+1} = a\,2^{n+1} + b(-2)^{n+1} = 2a\,2^n - 2b(-2)^n \longrightarrow (2)$$

$$y_{n+2} = a\,2^{n+2} + b(-2)^{n+2}$$

$$= a\,2^n.4 + b(-2)^n.4 = 4a\,2^n + 4b(-2)^n \longrightarrow (3)$$

Eliminating $a(2^n)$ & $b(-2)^n$ from (1), (2) & (3),

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 2 & -2 \\ y_{n+2} & 4 & 4 \end{vmatrix} = 0$$

$$y_n(8+8) - 1(4y_{n+1} + 2y_{n+2}) + 1(4y_{n+1} - 2y_{n+2}) = 0$$

$$16y_n - 4y_{n+1} - 2y_{n+2} + 4y_{n+1} - 2y_{n+2} = 0$$

$$16y_n - 4y_{n+2} = 0$$

$$y_{n+2} - 4y_n = 0.$$

(3) Derive the difference equation from $y_n = (A+Bn)2^n$

Soln:

$$y_n = (A+Bn)2^n = A2^n + Bn.2^n \longrightarrow (1)$$

$$y_{n+1} = A2^{n+1} + B(n+1)2^{n+1}$$

$$= 2A2^n + 2B(n+1)2^n \longrightarrow (2)$$

$$y_{n+2} = A2^{n+2} + B(n+2)2^{n+2}$$

$$= 4A2^n + 4B(n+2)2^n \longrightarrow (3)$$

Eliminating $A\cdot 2^n$ and $B\cdot 2^n$ from (1), (2) & (3),

$$\begin{vmatrix} y_n & 1 & n \\ y_{n+1} & 2 & 2(n+1) \\ y_{n+2} & 4 & 4(n+2) \end{vmatrix} = 0$$

$$y_n[8(n+2)-8(n+1)]-1[4(n+2)y_{n+1}-2(n+1)y_{n+2}]$$
$$+n[4y_{n+1}-2y_{n+2}]=0$$

$$y_n[8n+16-8n-8]-[(4n+8)y_{n+1}-(2n+2)y_{n+2}]$$
$$+4ny_{n+1}-2ny_{n+2}=0$$

$$8y_n-(4n+8)y_{n+1}+(2n+2)y_{n+2}+4ny_{n+1}-2ny_{n+2}=0$$

$$2y_{n+2}-8y_{n+1}+8y_n=0$$

$$y_{n+2}-4y_{n+1}+4y_n=0$$