

2. The differential equation of a system⁽⁵⁾ is given as $\frac{d^2}{dt^2} y(t) + 3\frac{d}{dt} y(t) + 2y(t) = x(t)$ with initial conditions $y(0^+) = 3$, $y'(0^+) = -5$. Determine the O/P for the i/p $x(t) = 2u(t)$.

$$\cancel{s^2 y(s) + 3s y(s) + 2y(s) = x(s)}$$

$$s^2 y(s) - s y(0^-) - y'(0^-) + 3[s y(s) - y(0^-)] + 2y(s) = x(s)$$

$$s^2 y(s) - 3s + 5 + 3[s y(s) - 3] + 2y(s) = x(s)$$

$$s^2 y(s) - 3s + 5 + 3s y(s) - 9 + 2y(s) = x(s)$$

$$y(s) [s^2 + 3s + 2] - 3s - 4 + 5 = x(s)$$

$$y(s) [s^2 + 3s + 2] - 3s - 4 = \frac{2}{s}$$

$$y(s) [s^2 + 3s + 2] = \frac{2}{s} + (3s + 4)$$

$$y(s) [s^2 + 3s + 2] = \frac{2 + 3s^2 + 4s}{s(s+1)(s+2)}$$

$$y(s) = \frac{3s^2 + 4s + 2}{s(s+1)(s+2)}$$

$$3s^2 + 4s + 2 = A(s+1)(s+2) + Bs(s+2) + Cs(s+1)$$

put $s=0$, $2 = 2A \Rightarrow A = 1$

$s=-1$, $1 = -B \Rightarrow B = -1$

$s=-2$, $6 = +2C \Rightarrow C = 3$

$$y(s) = \frac{1}{s} - \frac{1}{s+1} + \frac{3}{s+2}$$

$$= \mathcal{L}^{-1}\left[\frac{1}{s}\right] - \mathcal{L}^{-1}\left[\frac{1}{s+1}\right] + 3\mathcal{L}^{-1}\left[\frac{1}{s+2}\right]$$

$$y(t) = u(t) - e^{-t}u(t) + 3e^{-2t}u(t)$$

$$\textcircled{3} \quad \frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - 2y(t) = x(t)$$

$$s^2 y(s) - s y(s) - 2y(s) = x(s)$$

$$(s^2 - s - 2) y(s) = x(s)$$

$$H(s) = \frac{y(s)}{x(s)} = \frac{1}{s^2 - s - 2} = \frac{1}{(s-2)(s+1)}$$

$$\frac{1}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1} = A(s+1) + B(s-2)$$

put $s = 2$

$$1 = 3A \Rightarrow \boxed{A = 1/3}$$

$s = -1$

$$1 = -3B \Rightarrow \boxed{B = -1/3}$$

$$H(s) = \frac{1}{3(s-2)} - \frac{1}{3(s+1)}$$

$$= \frac{1}{3} L^{-1} \left[\frac{1}{s-2} \right] - \frac{1}{3} L^{-1} \left[\frac{1}{s+1} \right]$$

$$\boxed{h(t) = \frac{1}{3} e^{2t} u(t) - \frac{1}{3} e^{-t} u(t)}$$

$\textcircled{4}$ $H(s) = \frac{s}{s^2 + 5s + 6}$ and $x(t) = e^{-t} u(t)$.
Determine the o/p assuming zero initial conditions.

$$H(s) = \frac{y(s)}{x(s)} = \frac{s}{s^2 + 5s + 6}$$

$$x(s) = \frac{1}{s+1}$$

$$y(s) = \frac{s}{(s+2)(s+3)} \cdot \frac{1}{s+1}$$

$$y(s) = \frac{s}{(s+2)(s+3)(s+1)}$$

$$s = A(s+3)(s+1) + B(s+2)(s+1) + C(s+2)(s+3)$$