Impulse response

$$H(S) = \frac{S+2}{S^2+4S+3}$$

$$\frac{S+2}{(S+3)(S+1)} = \frac{A}{S+3} + \frac{B}{S+1}$$

$$S+2 = A(S+1) + B(S+3)$$

put $S=-1$ $\quad 1 = 2B \quad \boxed{B = 1/2}$

put $S=-3$ $\quad -1 = -2A \quad \boxed{A = 1/2}$

$$H(S) = \frac{1}{2(S+3)} + \frac{1}{2(S+1)}$$

$$h(t) = \frac{1}{2} L^{-1}\left[\frac{1}{S+3}\right] + \frac{1}{2} L^{-1}\left[\frac{1}{S+1}\right]$$

$$\boxed{h(t) = \frac{1}{2} e^{-3t} u(t) + \frac{1}{2} e^{-t} u(t)}$$

(5)

2. The Differential equation of a system is given as $\frac{d^2}{dt^2} y(t) + 3\frac{d}{dt} y(t) + 2y(t) = x(t)$ with initial conditions $y(0^+) = 3$, $y'(0^+) = -5$. Determine the o/p for the i/p $x(t) = 2.u(t)$

$$\cancel{s^2 \cdot y(s) + 3s\, y(s) + 2\, y(s) = x(s)}$$

$$s^2 \cdot y(s) - s\, y(0^-) - y'(0^-) + 3[s\,y(s) - y(0^-)] + 2y(s) = x(s)$$

$$s^2 \cdot y(s) - 3s + 5 + 3[s\,y(s) - 3] + 2\,y(s) = x(s)$$

$$s^2 y(s) - 3s + 5 + 3s\,y(s) - 9 + 2\,y(s) = x(s)$$

$$y(s)[s^2 + 3s + 2] - 3s - 9 + 5 = x(s)$$

$$y(s)[s^2 + 3s + 2] - 3s - 4 = \frac{2}{s}$$

$$y(s)[s^2 + 3s + 2] = \frac{2}{s} + (3s + 4)$$

$$y(s)[s^2 + 3s + 2] = \frac{2 + 3s^2 + 4s}{s}$$

$$\boxed{y(s) = \frac{3s^2 + 4s + 2}{s(s+1)(s+2)}}$$

$$3s^2 + 4s + 2 = A(s+1)(s+2) + Bs(s+2) + Cs(s+1)$$

put $s = 0$, $2 = 2A \Rightarrow \boxed{A = 1}$

$s = -1$, $1 = -B \Rightarrow \boxed{B = -1}$

$s = -2$, $6 = +2C \Rightarrow \boxed{C = 3}$

$$y(s) = \frac{1}{s} - \frac{1}{s+1} + \frac{3}{s+2}$$

$$= L^{-1}[1/s] - L^{-1}\left[\frac{1}{s+1}\right] + 3L^{-1}\left[\frac{1}{s+2}\right]$$

$$\boxed{y(t) = u(t) - e^{-t}u(t) + 3e^{-2t}u(t)}$$

(3) $\frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - 2y(t) = x(t)$

$$s^2 y(s) - s y(s) - 2y(s) = x(s)$$

$$(s^2 - s - 2)\, y(s) = x(s)$$

$$H(s) = \frac{y(s)}{x(s)} = \frac{1}{s^2 - s - 2} = \frac{1}{(s-2)(s+1)}$$

$$\frac{1}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1} = A(s+1) + B(s-2)$$

put $s = 2$

$$1 = 3A \Rightarrow \boxed{A = 1/3}$$

$s = -1$

$$1 = -3B \Rightarrow \boxed{B = -1/3}$$

$$H(s) = \frac{1}{3(s-2)} - \frac{1}{3(s+1)}$$

$$= \frac{1}{3} L^{-1}\left[\frac{1}{s-2}\right] - \frac{1}{3} L^{-1}\left[\frac{1}{s+1}\right]$$

$$\boxed{h(t) = \frac{1}{3} e^{2t} u(t) - \frac{1}{3} e^{-t} u(t)}$$

(4) $H(s) = \frac{s}{s^2 + 5s + 6}$ and $x(t) = e^{-t} u(t)$. Determine the o/p assuming zero initial conditions.

$$H(s) = \frac{y(s)}{x(s)} = \frac{s}{s^2 + 5s + 6}, \quad x(s) = \frac{1}{s+1}$$

$$y(s) = \frac{s}{(s+2)(s+3)} \cdot \frac{1}{s+1}$$

$$y(s) = \frac{s}{(s+2)(s+3)(s+1)}$$

$$s = A(s+3)(s+1) + B(s+2)(s+1) + C(s+2)(s+3)$$

put $s = -3$, $-3 = 2B \Rightarrow B = -2/3$ (6)

$s = -2$ $\quad -2 = -A \Rightarrow A = 1/2$

$s = -1$, $\quad -1 = 2C \Rightarrow C = -1/2$

$$y(s) = \frac{1}{2(s+2)} - \frac{2}{3(s+3)} - \frac{1}{2(s+1)}$$

$$y(t) = \frac{1}{2} e^{-2t} u(t) - \frac{2}{3} e^{-3t} u(t) - \frac{1}{2} e^{-t} u(t)$$

5. Find the impulse response of the system

$$RC \frac{d}{dt} y(t) + y(t) = x(t)$$

$$RC\, s \cdot y(s) + y(s) = x(s)$$

$$H(s) = \frac{y(s)}{x(s)} = \frac{1}{RCs+1}$$

$$y(t) = L^{-1}\left[\frac{1}{RCs+1}\right]$$

$$= \frac{1}{RC} L^{-1}\left[\frac{1}{s + 1/RC}\right]$$

$$\therefore y(t) = \frac{1}{RC} e^{-t/RC} . u(t)$$

6. Find the o/p of the system

$h(t) = u(t)$, $\quad x(t) = e^{-2t} . u(t)$

$H(s) = 1/s$, $\quad X(s) = \frac{1}{s+2}$

$$y(s) = H(s) . X(s)$$

$$= \frac{1}{s} \cdot \frac{1}{s+2} = \frac{A}{s} + \frac{B}{s+2} \Rightarrow A(s+2) + B(s)$$

put $s = 0$ $\quad 2A = 1 \Rightarrow \boxed{A = 1/2}$

$s = -2$ $\quad -2B = 1 \Rightarrow \boxed{B = -1/2}$

$$y(s) = \frac{1}{2s} - \frac{1}{2(s+2)}$$

$$= \frac{1}{2} L^{-1}[1/s] - 1/2\, L^{-1}\left[\frac{1}{s+2}\right]$$

$$y(t) = 1/2\, u(t) - 1/2\, e^{-2t} u(t).$$