

SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution) Coimbatore-641035.



UNIT 5– LATTICES AND BOOLEAN ALGEBRA

Properties of lattices

Pelopertres of Lattices: Let (L, A, V) be a given lattice. Then for any a, b, CEL. D. Idempotent law ana = a and ava = a 2). commutative law and = bra and avb = bva 3). Associative law (anb) AC = an (bac) and (avb) VC = av (bvc) A). ABSOSPTUDD Jaw $Q_{\Lambda}(QVD) = Q$ and $Q_{\Lambda}(Q\Lambda D) = Q$ Peoof: 1). Idempotent law: Now ava = LUB(a, a) = LUB(a) = aava = a $a \wedge a = GLB(a, a) = GLB(a) = Q$ and ana =a 2). Commutative law: NOW avb = LUB(a, b) = LUB(b, a) = bvaand and = GLB(a, b) = GLB(b, a) = bra 3). Associative : Let av(bvc) = d -> (1) QVB) VC = E -> (2) (1) > d 92 LUB of (9, byc) => dra and drbvc -> (3) WHT BUC & LUB OF (B,C) brezbard brezedra dyb dre > (5)



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UNIT 5- LATTICES AND BOOLEAN ALGEBRA **Properties of lattices** (5) = d & an UB of (a, b) and d > c d > avb and d > c , d is an UB of (avb, c) Sance e 92 LUB of (avb, c) dre My ezd · e=d. = av(bvc) = (avb) vc 4). Absorption law: av(anb) = a Sance and B GLB of Fa, 67 $anb \leq a \rightarrow ()$ obviously a za -> (2) (1) and (2), $av(ahb) \leq a \rightarrow (3)$ By defn, of LUB, we've $a \leq av(a \wedge b) \rightarrow (4)$ av(a,b) = a. Theorem: 1 Isotoricity of law of peoperty Let (L, A, V) be a gvz. lattace for any a, b, c $\in L$. Then prove that bec $\Rightarrow I$. a, b \in and a). avb \in avc Ploop: GARVED BEC GLBEB, c) = bAC = b (SANCE be of brc= c LOB (b, c) = bVC = c (SANCE be of brc= c To plove : 1) and E anc. It is chough to prove that GILB (and, anc) = and ie, (anb) 1 (anc) = anb



NOW

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(and) n(anc) = an(bna) nc = a 1(a1b) 1 C = (ana)n (BAC) = ar (brc) - anb · · and eand 2). axb ≤ avc It is enough to peove LUB (avb, avc) = avc (avb)v(avc) = avcNow (avb) v (avc) = av (bva) VC = av (avb) vc = (ava) V (bvc) = a v (bvc) = avc · avb ≤ ave Theorem: & DPStopbuttere Prequalito Let (LSA, V) be a given lattice for any a, b, CEL, the following enequality bolds. D. av(BAC) ≤ (avb) A (axc) ii). an (bvc) ≥ (anb) v(anc) Proof: D. ar(bAC) = [avb) A (avc) From the defn of LUB, It & obvious that azavb \rightarrow (1) and bac Eb Earb => bAC ≤ avb > (2)



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from (1) and (2), avb is an upper bound of Ja, bacy Hence avb>av(bAc) -> (A) flom the defe of LUB, it is obvious that $a \leq avc \rightarrow (3)$ and bACECEAVC => BACE avc -> (4) From (3) and (4), ave is an UB of Za, breg Hence $avc \geq av(bAC) \rightarrow (B)$ FLOOM (A) and (B), wo've av (BAC) & ALB of javb, avc 3. : av (brc) = (avb) r (avc) Hence ploved (i). ii). $an(Byc) \ge (anb) \vee (anc)$. WHT azand -> [] and BYC > b > alb BYC ZAAD ~ > (2) From (1) and (2), and is an LB of 29, by c3 Hence and < an(bvc) ->(c) WKT azanc -> (3) and brezezanc From (3) & (4), and is an LB of Za, breg Hence anc = an (bvc) -> (D) From (c) and (d), we've an(bvc) is an UB of Earb, ancy. ··· AN(Brc) ≥ (AND) V(ANC) Hence proved (i).