SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF MECHANICAL ENGINEERING

R2019 – FLUID MECHANICS AND MACHINERY

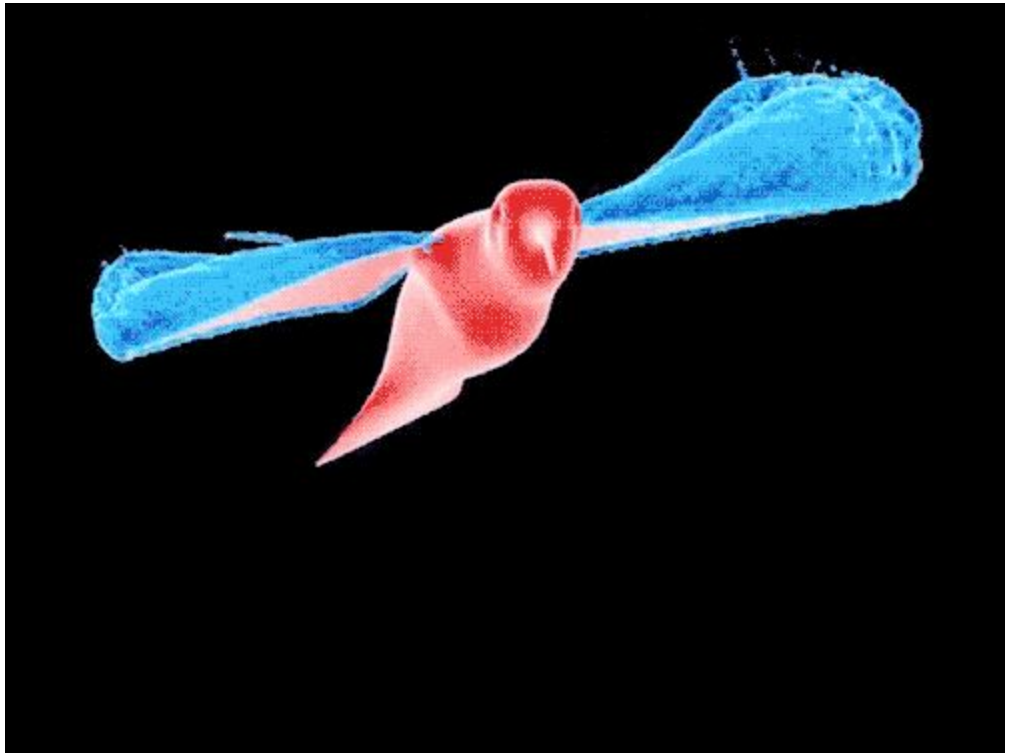
II YEAR III SEM

UNIT 3 – DIMENSIONAL ANALYSIS AND SIMILITUDE

TOPIC 2 – DIMENSIONAL ANALYSIS METHODS

CONTENT

- RALEIGH'S METHOD
- BUCKINGHAM PI THEOREM
- ASSESSMENT(MCQ)



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RALEIGH'S METHOD

Rayleigh's Method: In this method the functional relationship of some variables is expressed in the form of an exponential equation, which must be dimensionally homogenous. If 'Y' is function of independent variables X_1, X_2, X_3 . . . etc then functional relationship may be written as

$$Y = f(X_1, X_2, X_3 \ldots\ldots)$$

Steps:

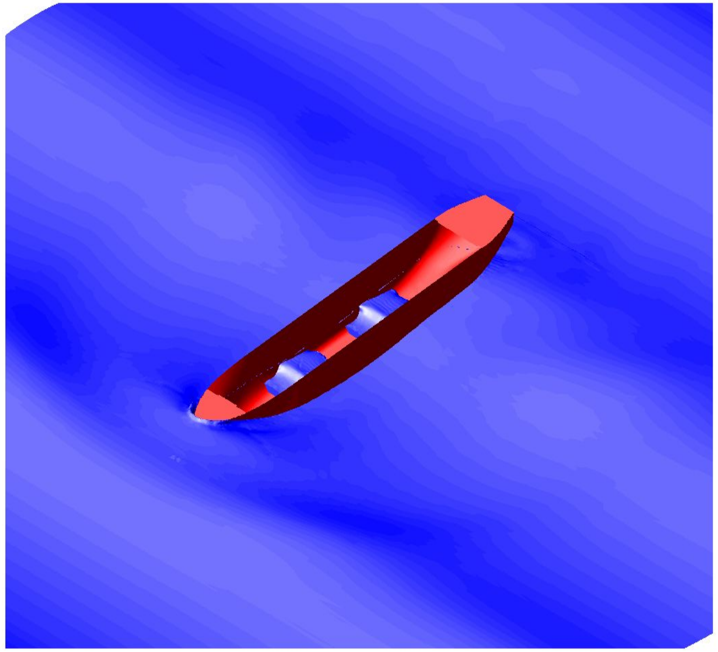
1) Write the functional relationship with all given data.

2) Write equation in terms of constant with exponents (i.e. Power) a, b, c.....

$$Y = K \cdot X_1^a \cdot X_2^b \cdot X_3^c \cdot \ldots\ldots$$

1) Find the value of a, b, c... With the help of dimensional homogeneity.

2) Substitute the values of the exponents in the main equation and simplify it.



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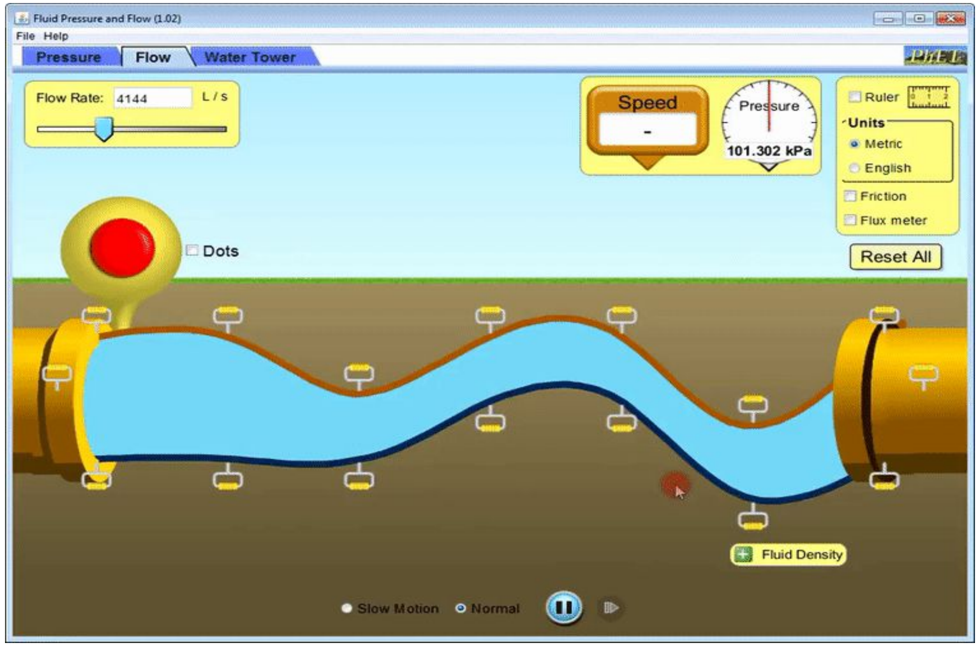
RALEIGH'S METHOD

Derivation and Numerical



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BUCKINGHAM Pi THEOREM

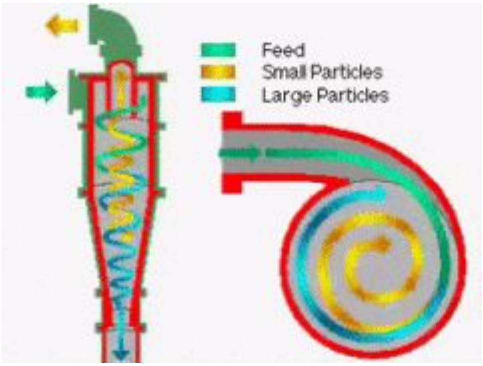
Buckingham's π-Theorem: If an equation involving k variables is dimensionally homogeneous, it can be reduced to a relationship among k-r independent dimensionless products, where r is the minimum number of reference or primary dimensions required to describe the variables.

- Given a physical problem in which the dependent variable is a function of k-1 numbers of independent variables

  $u_1 = f(u_2, u_3, \ldots, u_k)$

- Mathematically, we can express the functional relationship in the equivalent form $g(u_1, u_2, u_3, \ldots, u_k) = 0$; where g is an unspecified function different from 'f'

- The Buckingham Pi theorem states that: Given a relation among k variables of the form $g(u_1, u_2, u_3, \ldots, u_k) = 0$



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- The k variables may be grouped into k-r independent dimensionless products, or Π terms, expressible in functional form by

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots, \Pi_{k-r}) \quad \text{or} \quad \phi(\Pi_1, \Pi_2, \Pi_3, \ldots, \Pi_{k-r}) = 0$$

BUCKINGHAM Pi THEOREM

<u>Buckingham's π-Theorem (continue):</u>

- The number r is usually (but not always) equal to the minimum number of independent dimensions required to specify the dimensions of all the parameters. Usually the reference dimensions required to describe the variables will be the basic dimensions M, L, and T or F, L, and T

- The theorem does not predict the functional form of ϕ . The functional relation among the independent dimensionless products Π must be determined experimentally

- The k-r dimensionless products Π terms obtained from the procedure are independent

- A Π term is not independent if it can be obtained from a product or quotient of the other dimensionless products (or Π term) of the problem. For example, if

$$\Pi_5 = \frac{2\Pi_1}{\Pi_2\Pi_3} \quad \text{or} \quad \Pi_6 = \frac{\Pi_1^{3/4}}{\Pi_3^{2}}$$

then, neither Π_5 nor Π_6 is independent of the other dimensionless products or dimensionless groups

BUCKINGHAM Pi THEOREM

<u>Steps to Find PI (π) Terms:</u>

1. List all the variables: Assume ‘k’ is the number of variables in the problem or phenomenon

2. Express each of the variables in terms of basic dimensions. Find the minimum number of reference dimensions or primary dimension. Assume ‘r’ is the minimum number of primary dimensions in MLT or FLT system

3. Determine the required number of ‘π’ (PI) term. The number of ‘π’ (PI) term will be ‘k-r’

4. Select a number of repeating variables, where the number required is equal to the number of reference dimensions. Then select a set of r dimensional variables that includes all the primary dimensions (repeating variables) . These repeating variables will all be combined with each (or one) of the remaining parameters. No repeating variables should have dimensions that are power of the dimensions of another repeating variable. Normally, one from geometry, one from fluid property and one from flow characteristics

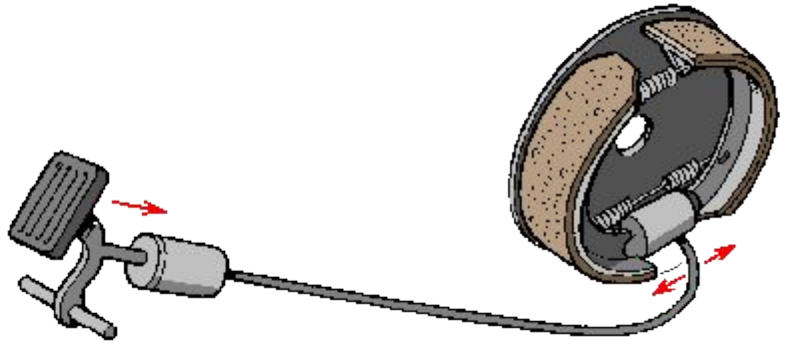
BUCKINGHAM Pi THEOREM

Steps to Find PI (π) Terms:

5. Form a pi term by multiplying one of the non-repeating variables by the product of the repeating variables, each raised to an exponent that will make the combination dimensionless. There will be k –r equations for **π** (PI) terms
6. Check all the resulting **π (PI)** terms to make sure they are dimensionless.
7. Express the final form as a relationship among the **π (PI)** terms, and think about what is means. Express the result of the dimensional analysis

Question :Why do we go for Pi theorem?

$$\Pi_1 = \phi(\Pi_2, \Pi_3,,,, \Pi_{k-r})$$



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BUCKINGHAM Pi THEOREM

Problem-1:

Pressure drop per unit length depends on FOUR variables: Pipe diameter (D); flow velocity (V); fluid density (ρ) ; fluid viscosity (μ). Show that by Buckingham Π (PI) theorem by both FLT and MLT system.

$$\frac{D\Delta p_\ell}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$

Solution:

(1). First step to find all variables

$$(\Delta p_\ell, D, \mu, \rho, \text{ and } V)$$

(2). Find number of variables, K = 5 and

(3). Need to find minimum no of primary dimensions for each variables either MLT or FLT system. (Let assume in FLT system).

Note:

Δp_ℓ = Pressure drop per unit length of pipe

BUCKINGHAM Pi THEOREM - NUMERICAL

Problem-1(continue):

Solution:

(3). Need to find minimum no of primary dimensions for each variables either MLT or FLT system. (Let assume in FLT system).

$$\Delta p_\ell = f(D, \rho, \mu, V)$$

FLT:

$$\Delta p_\ell = FL^{-3} \qquad D = L \qquad \rho = FL^{-4}T^2$$

$$\mu = FL^{-2}T \qquad V = LT^{-1}$$

(4). Find minimum no of primary dimensions is required to express all terms.

Here minimum of primary dimensions required is 3 i.e. r = 3.

BUCKINGHAM Pi THEOREM - NUMERICAL

Problem-1(continue):

Solution:

(4). Find minimum no of primary dimensions is required to express all terms.

Here minimum of primary dimensions required is 3 i.e. $r = 3$.

(5). Number of Π (PI) terms will be $k-r = 5-3 = 2$. So we need two Π terms.

(6). Now find no of repeated variables and it will be as same number of r i.e. 3.

(7). Select repeated variables (at least one from geometry, one from fluid property and one from flow characteristics). Assume repeated variable will be ρ (density), V (velocity) and D (diameter of pipe).

(8). Now make each Π terms with three repeated variables and one non repeated variable. Need to put exponent (power on each repeated variable).

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c \qquad \Pi_2 = \mu D^a V^b \rho^c$$

BUCKINGHAM Pi THEOREM - NUMERICAL

Problem-1(continue):

Solution:

(8). Now make each Π terms with three repeated variables and one non repeated variable. Need to put exponent (power on each repeated variable).

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c \qquad \Pi_2 = \mu D^a V^b \rho^c$$

(9). Substitute FLT for each variable in each Π equation and find value of a, b , c.

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c$$

$$(FL^{-3})(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$F: 1 + c = 0$$

$$L: -3 + a + b - 4c = 0$$

$$T: -b + 2c = 0$$

$$a = 1, b = -2, c = -1$$

$$\Pi_1 = \frac{\Delta p_\ell D}{\rho V^2}$$

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BUCKINGHAM Pi THEOREM - NUMERICAL

Problem-1(continue):

Solution:

(9).

$$\Pi_2 = \mu D^a V^b \rho^c$$

$$(FL^{-2}T)(L)^a (LT^{-1})^b (FL^{-4}T^2)^c \doteq F^0 L^0 T^0$$

$$F: 1 + c = 0$$

$$L: -2 + a + b - 4c = 0$$

$$T: 1 - b + 2c = 0$$

$$a = -1, b = -1, c = -1$$

Substitute the value of a, b, c in second equation:

$$\Pi_2 = \frac{\mu}{DV\rho}$$

(10). Check each Π term to make sure , we didn't make any mistake. Each Π term should dimensionless.

(11). Finally find expression by using formula $\Pi_1 = \phi(\Pi_2, \Pi_3,,,, \Pi_{k-r})$

$$\Pi_1 = \phi(\Pi_2) \qquad \frac{\Delta p_\ell D}{\rho V^2} = \phi\left(\frac{\mu}{DV\rho}\right)$$

BUCKINGHAM Pi THEOREM

Derivation and Numerical

ASSESSMENT

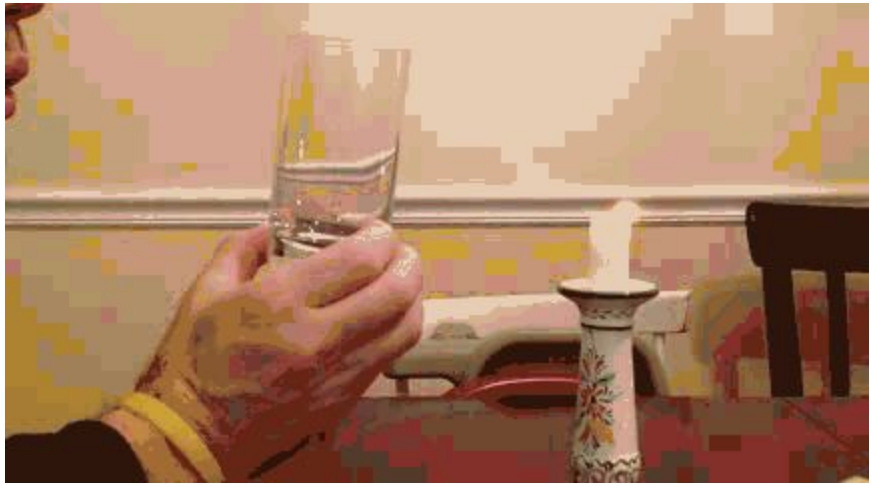
1. **What is the mathematical technique used to predict physical parameters?**

   a) Combustion analysis

   b) Pressure analysis

   c) Dimensional analysis

   d) Temperature analysis



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Answer: c

ASSESSMENT

2 Which among the following is the correct format for Rayleigh's method?

a) D = f(l,ρ,μV,g)

b) D = (l,ρ,μV,g)

c) D = f

d) D = f(lpv)

Answer: a

ASSESSMENT

3. Why does Rayleigh’s method have limitations?

a) To many variables

b) Format

c) Exponents in between variables

d) Many exponents

Answer: c

ASSESSMENT

4. What is a model analysis?

a) A small-scale replica

b) Actual structure

c) Theory structure.

d) Adopted structure

Answer: a

ASSESSMENT

5.Advantage of a model analysis is__________

a) Performance cannot be predicted

b) The relationships between the variable cannot be obtained

c) Shear stress to thermal diffusivity

d) Alternative designs can be predicted

Answer: d

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